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Faculty of Mathematics, University of Belgrade, Serbia
Faculty of Science, University of Kragujevac, Serbia
Mathematical Institute of the Serbian Academy of Sciences and Arts

XXII GEOMETRICAL SEMINAR

Book of Abstracts

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PREFACE

The international conference XXII Geometrical Seminar 2024, is to be held at Vrnjačka Banja, Serbia from May 26th to 31st, 2024.

This Book of abstracts is collection of abstracts of talks to be presented at the conference from different geometric topics. Participants will have 15 and 30 minutes talks. The XXII Geometrical Seminar has more than 100 participants from all over the world. This meeting is bringing together mathematicians, physicists and engineers interested in Geometry and its applications. The aim of the Geometrical Seminar is to enable researches to give lectures on new results, exchange ideas, problems and conjectures. This issue contains abstracts of talks to be presented at the XXII Geometrical Seminar. The abstracts were accepted for presentation after having been subjected to the usual strict reviewing process of the conference committee. The editors thank the members of the Committees of Geometrical Seminar and to the Faculty of Sciences and Mathematics, Niš, Faculty of Mathematics, Belgrade, to the Faculty of Science, Kragujevac and Mathematical Institute SASA for their great effort in organizing this conference.

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Integrable geodesic flows with rational first integrals

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We study integrable geodesic flows on 2-surfaces. In most known examples, the additional integrals are polynomial in momenta. Polynomial integrals of small degrees are well- studied and classified in both local and global aspect of the problem.

On the other hand, as proved in [1], there exist local Riemannian metrics on 2-surfaces with integrable geodesic flows such that additional integrals are rational in momenta with any given degrees of a numerator and a denominator. However, constructing such examples in an explicit form turned out to be a very difficult problem. In this talk we will describe various methods and approaches which allowed us to construct rich families of 2-dimensional Riemannian metrics with integrable geodesic flows admitting rational first integrals (see [2], [3]).

The talk is based on joint work with V. Shubin.

References

[1] V. V. Kozlov, *On rational integrals of geodesic flows*, Regul. Chaotic Dyn., **19**, 601–606 (2014).

[2] S. Agapov and V. Shubin, *Rational integrals of 2-dimensional geodesic flows: New examples*, Journ. Geom. Phys., **170**, 104389 (2021).

[3] S. Agapov and V.Shubin, *New examples of non-polynomial integrals of two- dimensional geodesic flows*, J. Phys. A: Math. Theor., **57**, 015204 (2024).

Hopf Bundle, Listing's Law and Saccades

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Our eyes continually move. Even while we fix our gaze on a still object, they participate in fixational eye movements – tremor, drift and microsaccades. The classical experiments by A. Yarbus show that compensation of eye movements leads to loss of vision in 2-3 sec.

The configuration space of the eye is described by the Listing's law. We propose an interpretation of the Listing's law in terms of the Hopf bundle as a Listing's hemisphere and use it for description of the saccades and saccadic cycles. We discuss the role of saccades and drift in vision. Roughly speaking, vision is the result of the interaction of information about light entering the retina and recorded by photoreceptors and oculomotor information about the eye position after remapping that accompanies a saccade.

Neutrosophic Touchard Polynomials for Subclass of Analytic Functions

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By using the generalization of the neutrosophic Touchard Polynomials, we introduce a new subclass of analytic functions defined in the open unit disk. We then apply the q-Touchard Polynomials to investigate the estimates for the Taylor coefficients and Fekete-Szeg type inequalities of the functions belonging to this new subclass. In addition, we consider several corollaries and the consequences of the results presented in this paper. The neutrosophic Touchard Polynomials is expected to be significant in a number of areas of mathematics, science, and technology.

Convergence theorems for sequences of operator valued functions

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We consider spaces of integrable operator valued functions, in the sense of Gelfand. Analogues of Dominated Convergence Theorem and of Vital Convergence Theorem are obtained, and a converse of Vitali Convergence Theorem is proved. Some of the results are specific for Hilbert space operators, namely those which rely on the notion of absolute value of operator. We note that earlier research centered on a single integrable operator valued functions and Radon - Nikodym properties. This is a joint work with Mihailo Krstic.

On a coarse geometric approach to operator algebras

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Coarse geometry is an interesting part of metric geometry, working with mappings that are not necessarily continuous and only need to be checked. In other words, there are such positive constants A,B that hold for metric spaces (e.g. graphs).

$$\forall x, y \in M_1 : \frac{1}{A}\, \rho^1(x,y) - B \leq \rho^2(f(x), f(y)) \leq A\, \rho^1(x,y) + B;$$

Here $f : M_1 \to M_2$ is a mapping between metric spaces M_1, M_2. We will show that for graphs generated by the action of a group on itself (a well-known example are Cayley graphs), operator algebras are generated. For Cayley graphs this is the set of Fox derivations and for conjugacy diagrams we get a classical "Leibniz" derivation.

It is shown that the family of operators is coarsely invariant.

Pointwise rectifying submanifolds and anti-torqued vector fields

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In this talk, we introduce a notion of pointwise rectifying submanifold in a Riemannian manifold with respect to a vector field called axis. We concern with the case that the axis is an anti-torqued vector field. Hence, we first find a necessary and sufficient condition for a Riemannian manifold to be endowed with an anti-torqued vector field. We then classify pointwise rectifying submanifolds when the axis is anti-torqued. Non-trivial examples are also provided.

This is a joint work with Adela Mihai and Cihan Özgür.

References

[1] B.-Y. Chen, *Differential geometry of rectifying submanifolds*, Int. Electron. J. Geom. 9(2) (2016), 18.

[2] B.-Y. Chen, *Rectifying submanifolds of Riemannian manifolds and torqued vector fields*, Kragujev. J. Math. 41 (2017), 93103.

[3] S. Deshmukh, I. Al-Dayel, D. M. Naik, *On an anti-torqued vector field on Riemannian*, Mathematics, 9 (2021), 2201.

Left-invariant f-structures on low-dimensional solvable Lie groups

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Metric f-structures by K.Yano ($f^3+f=0$) play a leading role in the generalized Hermitian geometry (V.F.Kirichenko, D.Blair, S.Salamon and others). The most interesting classes are Killing f-structures ($\nabla_X(f)(X)=0$), nearly Kähler f-structures ($\nabla_{fX}(f)(fX)=0$), generalized nearly Kähler f-structures ($f\nabla_{fX}(f)(fX)=0$) introduced since the 1990s by V.F.Kirichenko, A.Gritsans, V.Balashchenko. In the particular case $f=J$ ($J^2=-1$) all of them give the well-known nearly Kähler structures ($\nabla_X(J)(X)=0$). We also mention the class of Hermitian metric f-structures ($T(X,Y)=0$), where $T(X,Y)=\frac{1}{4}f(\nabla_{fX}(f)(fY)-\nabla_{f^2X}(f)(f^2Y))$ is the composition tensor (V.F.Kirichenko).

We concentrate on left-invariant metric f-structures on solvable Lie groups in dimension 3 and 4. Note that three- dimensional solvable Lie algebras were first classified by G.M.Mubarakzyanov in 1963. Using special left-invariant metric f-structures on all three-dimensional solvable Lie algebras we present the examples of all the classes in the generalized Hermitian geometry above mentioned. In particular, for the Lie algebras $[e_1,e_2]=\lambda e_2-e_3; [e_1,e_3]=e_2+\lambda e_3$ depending on parameter λ we define f-structure by the requirement $f(e_1)=0, f(e_2)=-e_3, f(e_3)=e_2$. It was proved that the left-invariant f-structure on the corresponding Lie group is a nearly Kähler f-structure for $\lambda=-1$ and a Hermitian f-structure for arbitrary λ. Among four-dimensional solvable Lie groups the oscillator group is of especial interest in geometry, control theory and other applications.

The corresponding Lie algebra is defined by the commutator relations $[e_2, e_3] = e_1, \quad [e_2, e_4] = -e_3, \quad [e_3, e_4] = e_2$. We completely investigate all basic left-invariant f-structures of rank 2 on this Lie group in the above sense.

This is a joint work with Victoria Kunitsa.

The mod p Buchstaber invariant

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In this talk, we present combinatorial and topological properties of the universal complexes $X(\mathbb{F}_p^n)$ and $K(\mathbb{F}_p^n)$ whose simplices are certain unimodular subsets of $\mathbb{F}_p^n$. We calculate their **f**-vectors and the bigraded Betti numbers of their Tor-algebras, show that they are shellable, and find their applications in toric topology and number theory. We showed that the Lusternick-Schnirelmann category of the moment angle complex of $X(\mathbb{F}_p^n)$ is n, provided p is an odd prime and the Lusternick-Schnirelmann category of the moment angle complex of $K(\mathbb{F}_p^n)$ is $[\frac{n}{2}]$. Based on the universal complexes, we introduce the Buchstaber invariant s_p for a prime number p. We investigate the mod p Buchstaber invariant of the skeleta of simplices for a prime number p and compare them for different values of p. For $p = 2$, the invariant is the real Buchstaber invariant. Our findings reveal that these values are generally distinct. Additionally, we determine or estimate the mod p Buchstaber invariants of some universal simplicial complexes $X(\mathbb{F}_p^n)$. The talk is based on joint research with Aleš Vavpetić and Aleksandar Vučić.

Almost complex manifolds with Norden metrics

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On an almost complex manifold, a Norden structure is a semi- Riemannian metric, with respect to which the almost complex structure is an anti-isometry. We construct here a class of almost complex manifolds with Norden metric on the total space of the cotangent bundle (of a manifold with a symmetric connection), endowed with a natural Riemann extension (which generalize the Riemann extension, introduced by Patterson, Walker). Then we give necessary and sufficient conditions to belong to certain classes of Ganchev-Borisov classification of almost complex manifolds with Norden metric. Some harmonicity problems are studied. At the end, we construct a class of hypercomplex Norden structures.

Remarks on weighted Einstein manifolds

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A Riemannian manifold (M, g) is endowed with a smooth density function f, and two real parameters m and μ, giving rise to a smooth metric measure space (M, g, f, m, μ). In order to analyze smooth metric measure spaces, it is useful to consider weighted objects such as the weighted Schouten tensor P_f^m or the weighted Weyl tensor W_f^m, which reflect different aspects of the influence of the density on the geometry of the underlying manifold.

Weighted Einstein manifolds are smooth metric measure spaces which satisfy $P_f^m = \lambda g$, for λ constant. This is a generalization of Riemannian Einstein manifolds. However, although usual Einstein metrics have harmonic Weyl tensor, this is no longer true in the weighted context. Motivated by this fact, we analyze which smooth metric measure spaces are weighted Einstein and have harmonic Weyl tensor (in a suitable weighted sense). First, we provide a local classification result showing that, under the harmonicity condition, the underlying manifold is Einstein or decomposes as a particular warped product. Moreover, if the manifold is complete, then we provide a global rigidity theorem which shows that it either is a weighted analogue of a space form, or it belongs to a particular family of Einstein warped products with $\lambda < 0$.

Great antipodal sets and applications

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Greatest antipodal sets were introduced in 1982 by Chen and Nagano. Since then some applications of greatest antipodal sets to several areas of mathematics have been established by various mathematicians. In this talk, I will discuss some recent applications of greatest antipodal sets.

The Willmore energy variations and energy efficient architecture

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Since the shape is an important feature of objects and can be immensely useful in characterizing objects, it should point out the shape analysis considering the variation of magnitudes that characterize the shape itself. One of the fundamental functionals that measure the bending of a surface is the Willmore energy. In view of the meaning of ruled surfaces in aesthetics, statics, scale and manufacturing technologies, we point out the possibility of a mathematical analysis in the case of infinitesimal deformations by considering the variations of the Willmore energy on Gaudi surface under infinitesimal bending in R^3. Application could be connected with energy efficient building design.

AMS Subj. Class.: 53A05, 53C45, 92C40, 51M15
Key words: Willmore energy, Gaudi surface, Infinitesimal bending, Variation, Energy efficient architecture.

Almost Kähler structures on complex hyperbolic space

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In this talk we consider Complex hyperbolic space as a solvable Lie group. Based on the recent classification of Riemannian left-invariant metrics [1] we describe all left-invariant almost Kähler structures on that group [2]. We calculate the canonical connection and obtain the characterization by its torsion tensor.

References

[1] A. Dekić, M. Babić, S. Vukmirović, *Classification of Left Invariant Riemannian metrics on Complex hyperbolic space*, Mediterr. J. Math., 19:232, (2022)

[2] A. Dekić, *Almost Kähler structures on complex hyperbolic space*, Filomat, Vol 37, No 25, (2023)

Curvature properties of pseudosymmetry type of some 2-quasi Einstein manifolds

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In this talk we present curvature properties of pseudosymmetry type of 2-quasi Einstein semi-Riemannian manifolds (M, g), $\dim M \geq 4$, with the Riemann-Christoffel curvature tensor R expressed by a linear combination of Kulkarni-Nomizu products $g \wedge g$, $g \wedge S$, $S \wedge S$ and $g \wedge S^2$, formed by the metric tensor g, the Ricci tensor S and its square S^2. Our talk is based on [1], [2], [3] and [4].

References

[1] R. Deszcz, M. Głogowska, M. Hotloś, M. Petrović-Torgašev and G. Zafindratafa, *A note on some generalized curvature tensor*, Int. Electron. J. Geom. 16 (1) (2023), 379–397.

[2] R. Deszcz, M. Głogowska, M. Hotloś, and K. Sawicz, *Hypersurfaces in space forms satisfying a particular Roter type equation*, arXiv: 2211.06700v3 [math.DG] 22 Nov 2023, 24 pp.

[3] R. Deszcz, M. Głogowska, J. Jełowicki, M. Petrović-Torgašev, G. Zafindratafa, *Curvature properties of pseudosymmetry type of some 2-quasi Einstein manifolds*, to appear.

[4] R. Deszcz, M. Głogowska, J. Jełowicki, G. Zafindratafa, *Curvature properties of some class of warped product manifolds*, Int. J. Geom. Meth. Modern Phys. 13 (2016), 1550135 (36 pages).

Remarks on weighted Einstein manifolds

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The Schwarzschild-de Sitter Metric of Nonlocal $\sqrt{dS}$ Gravity Abstract: Nonlocal de Sitter gravity model defined by the action

$$S = \frac{1}{16\pi G} \int \big(R - 2\Lambda + P(R)\ \mathcal{F}(\Box)\ Q(R)\big)\sqrt{-g}\ dx^4, \tag{1}$$

which we denote as $\sqrt{dS}$ gravity, contains an exact vacuum cosmological solution which mimics dark energy and dark matter and is in very good agreement with the standard model of cosmology. In this presentation we investigate how it works at galactic scale. We consider Schwarzschild-de Sitter metric of the $\sqrt{dS}$ gravity model. We present an approximative solution of linearized equation, which is related to space metric far from the massive body, where gravitational field is weak. The obtained solution is of particular interest for examining the possible role of non-local de Sitter gravity $\sqrt{dS}$ in describing the effects in galactic dynamics that are usually attributed to dark matter. The solution has been tested on the Milky Way and the spiral galaxy M33 and is in good agreement with observational measurements.

Nonlocal de Sitter Gravity $\sqrt{dS}$ and the Dark Side of the Universe

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I will present nonlocal de Sitter gravity model, denoted by $\sqrt{dS}$,

$$S = \frac{1}{16\pi G} \int d^4x \ \sqrt{-g} \ \big(R - 2\Lambda + \sqrt{R - 2\Lambda} \ \mathcal{F}(\Box) \ \sqrt{R - 2\Lambda}\big), \tag{2}$$

where Λ is the cosmological constant, $\Box$ is d'Alembert-Beltrami operator, and nonlocal operator $\mathcal{F}(\Box)$ has the following general form:

$$\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} \big(f_n \Box^n + f_{-n} \Box^{-n}\big). \tag{3}$$

This nonlocal model is unique compared to other non-local models and its properties will be pointed out.

Several exact vacuum cosmological solutions are obtained. One of these solutions is $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}$, and it mimics effects that are usually assigned to dark matter and dark energy. Obtained results are in a very good agreement with the standard model of cosmology. Some other solutions are examples of the nonsingular bounce ones in flat, closed and open universe. There are also singular and cyclic solutions. All these cosmological solutions are a result of nonlocality and do not exist in the local de Sitter case.

This talk is based on the joint research with Ivan Dimitrijevic, Jelena Stankovic and Zoran Rakic, and our common paper: JHEP 12 (2022) 054.

Finsler geodesic orbit metrics

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There are many results about geodesic orbit metrics in Riemannian geometry. Recently, the topic attracted attention also in Finsler geometry. Our approach to geodesic orbit metrics is through geodesic graphs, which are the generating sets for homogeneous geodesics. In Riemannian geometry the interesting cases are those which have nonlinear structure. Various special Finsler metrics arising from Riemannian geodesic orbit metrics modify the structure of geodesic graphs in particular way. We are able to prove that some of these Finsler metrics have also geodesic orbit property, for example the (alpha,beta) metrics. The examples of Finslerian geodesic graphs on nilmanifolds and on spheres will be presented. Part of the talk is based on the joint work with Teresa Arias-Marco.

On maps obtained by small perturbations of skew products

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We consider C^1-smooth maps obtained by small perturbations (in the C^1-norm) of C^1-smooth skew products on a compact cylinder. We prove the geometric integrability of these maps and construct the example of the family of such maps with the global attractor, which is one-dimensional ramified continuum with the set of the continuum cardinality of ramification points of the order 3.

This research is supported by Russian Science Foundation (RSF), grant 24-21-00242, https://rscf.ru/en/project/24-21-00242/.

Some Remarks on Quasi-para-Sasakian Manifolds

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In this talk, we give structure and curvature identities for quasi-para-Sasakian manifolds. We investigate D-homothetic deformations on quasi-para-Sasakian manifolds and we characterize them according to constant curvature. Finally, we give a method for constructing examples of three-dimensional conformally flat quasi-para-Sasakian manifolds.

Spectrum problem of Schrodinger operators related to Landau-Ginzburg models

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An LG model (M, f) is given by a noncompact complex manifold M and the holomorphic function f defined on it, which is an important model in string theory. Recently, LG models attracted more attention due to their important applications in mirror symmetry and complex geometry. Typical examples include Laurent polynomials defined on log Calabi-Yau manifolds, which are mirror to different Fano manifolds. The B-theory of a LG model is related to the Hodge theory of a twisted Laplacian operator, which is a form Schrodinger operator defined on the noncompact manifold M. This talk will concern the spectrum problem of this kind of Schrodinger operators. We will prove a fundamental spectrum theorem when (M, f) satisfies a strongly tame condition. This theorem can be applied to many LG models appearing in mirror symmetry.

Critical metrics for quadratic curvature functionals

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The purpose of this talk is to describe homogeneous critical metrics for quadratic curvature functionals in dimensions three and four. In addition to Einstein metrics, there are a number of non-symmetric examples which, in some cases, correspond to Ricci solitons.

Applications of algebraic topology to operator algebras: Homogeneous C*-algebras

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The class of commutative C*-algebras was fully described in the early days of the development of the theory of operator algebras: these are precisely algebras of continuous complex- valued functions on locally compact Hausdorff spaces that vanish at infinity. After commutative, the next simplest class are the homogeneous C*-algebras, i.e. C*-algebras whose all irreducible representations are of the same finite dimension. However, the problem of classification of such algebras becomes considerably more complicated due to certain topological phenomena which do not occur in the commutative case. In this talk we shall present the overview of homogeneous C*-algebras with an emphasis on some more recent results.

The Fermat-Torricelli problem in normed spaces

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The talk discusses a generalization of the classical Fermat- -Torricelli problem to normed spaces of arbitrary finite dimension.

The problem of finding a point that minimizes the sum of distances from it to a given set of points in a metric space was first mentioned in the 17th century. In 1643 Fermat posed a problem for three points on the Euclidean plane, and in the same century Torricelli proposed a solution to this problem. Since then, various generalizations of this problem have been considered. The problem was formulated for an arbitrary number of points, the dimension of the space, as well as the norm given in this space. The simplicity of the formulation allows us to consider the problem even in an arbitrary metric space.

The work describes the application of a geometric approach to the problem and presents some new results that are obtained in the framework of real finite-dimensional normed spaces, called Minkowski spaces.

The aim of this talk is to present necessary and sufficient conditions for the uniqueness of the solution of the Fermat- -Torricelli problem for any n points in a fixed space, and more precise conditions for normed planes and three- dimensional spaces.

In addition, examples of the application of the criterion in the norms given by regular polygons and regular polyhedra are given.

Weight systems of framed chord diagrams: the Circuit-Nullity formula and Lie algebras

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Chord diagrams and 4T-relations appear in studying Vassiliev invariants (finite-order invariants) of knots. The theory of Vassiliev invariants is tightly associated with the theory of finite-order J-invariants of planar curves that was developed by Arnold. In studying this theory, there arise the notions of framed chord diagram and framed 4T relation. Framed chord diagrams are also applied in describing the combinatorics of non-general position of Legendre knots in three-dimensional manifolds.

One of the main properties of chord diagrams is as follows: considering the formal linear combinations of chord diagrams modulo 4T-relations, we can rather simply define the product of these linear combination. As a result, we obtain the algebra of chord diagrams. According to the Vassiliev-Kontsevich theorem, weight systems, that is, linear functions on this algebra, lead to Vassiliev invariants of knots. However, in the case of framed chord diagrams, taking a connected sum is not a correct operation.

In view of this, there arises the problem of searching for invariants of framed chord diagrams that essentially distinguish chords of different framings. In our work we generalize the Bar-Natan construction of weight systems induced by Lie algebra representations to the case of framed chord diagrams. This is a joint work with I.M.Nikonov.

The Riemannian curvature identities on Almost Complex Calabi-Yau with torsion 6-manifold and generalized Ricci solitons

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Curvature properties of the torsion connection on an Almost complex CYT 6-manifold are investigated. It is observed that on a compact ACYT 6-manifold the Nijenhuis tensor is parallel with respect to the torsion connection. It is shown that a compact ACYT 6-manifold has closed torsion if and only if the Ricci tensor of the torsion connection is equal to the covariant derivative of the Lee form with respect to the torsion connection. It is proved that a compact ACYT 6-manifold with closed torsion is Ricci flat if and only if either the norm of the torsion is constant or the Riemannian scalar curvature is constant. It is observed that any compact ACYT 6-manifold with closed torsion 3-form is a generalized gradient Ricci soliton and this is equivalent to a certain vector field to be parallel with respect to the torsion connection. In particular, this vector field is an infinitesimal authomorphism of the $SU(3)$ structure.

It is also proved that on a compact ACYT 6-manifold the curvature of the torsion connection $R \in S^2\Lambda^2$ and has vanishing Ricci tensor if and only if the three-form torsion is parallel with respect to the Levi-Civita and to the torsion connection simultaneously. In particular, the conditions $R \in S^2\Lambda^2, Ric = 0$ are equivalent to the condition that the curvature of the torsion connection satisfies the Riemannian first Bianchi identity, i.e. it is Kähler-like. In this case the torsion 3-form is closed and co-closed.

In the complex case, it follows that the curvature of the Strominger-Bismut connection on a six dimensional CYT space satisfies the conditions $R \in S^2\Lambda^2, Ric = 0$ if and only if it is Kähler-like, therefore flat, and has parallel torsion with respect to both the Levi-Civita and Strominger-Bismut connections.

The research is partially supported by Contract KP-06-H72-1/05.12.2023 with the National Science Fund of Bulgaria and the National Science Fund of Bulgaria, National Scientific Program "VIHREN", Project KP-06-DV-7.

Polynomial integrability of sub-Riemannian geodesic flows on compact Lie groups

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We classify almost multiplicity free subgroups K of compact simple Lie groups G. The problem is related to the integrability of Riemannian and sub-Riemannian geodesic flows of left-invariant metrics defined by a specific extension of integrable systems from T^*K to T^*G.

We use the chains of subalgebras in order to construct integrable nonholonomic and sub-Riemannian flows with left-invariant metrics and left-invariant nonholonomic distributions on compact Lie groups. While the nonholonomic problem is not Hamiltonian, the sub-Riemannian is.

The talk is based on the paper *Almost multiplicity free subgroups of compact Lie groups and polynomial integrability of sub-Riemannian geodesic flows* by Božidar Jovanović, Tijana Šukilović, Srdjan Vukmirović [Letters in Mathematical Physics, 114 (1) Article no. 14, 2024].

Persistent homology theory and toric topology

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Toric topology is a new rapidly developing area of Mathematics in the crossroads of algebraic geometry, homotopy theory, equivariant topology, homological algebra and combinatorics. The main object of study in toric topology is a polyhedral product.

Given any topological pair (X, A) and simplicial complex K on $[m] := \{1, 2, \ldots, m\}$, the polyhedral product $(X, A)^K$ is a subset in X^m s.t. an m-tuple $(x_1, \ldots, x_m)$ is in $(X, A)^K$ iff there is a $\sigma \in K$: $i \notin \sigma \Rightarrow x_i \in A$. Our research is based on the use of the moment-angle complex $\mathcal{Z}_K := (D^2, S^1)^K$ of K.

Various applications of toric topology are related with the theorem by Baskakov, Buchstaber and Panov stating that cohomology $H^*(\mathcal{Z}_K; \Bbbk)$ of $\mathcal{Z}_K$ with coefficients in a PID with unit $\Bbbk$ is isomorphic as a $\Bbbk$-algebra to the Tor-algebra $\mathrm{Tor}_{\Bbbk[m]}(\Bbbk[K], \Bbbk)$ of K (homology of the Koszul complex of K), where $\Bbbk[m] := \Bbbk[v_1, \ldots, v_m]$ and the Stanley-Reisner ring $\Bbbk[K]$ is its quotient by the ideal generated by all monomials corresponding to the non- simplices in K.

In this talk we shall discuss certain key results obtained recently in the framework of the persistent double homology theory.

Namely, we shall introduce three equivalent definitions (geometrical, algebraic and topological ones) of a new differential acting on the singular (co)chain complex of $\mathcal{Z}_K$ and define the double cohomology of $\mathcal{Z}_K$ w.r.t. the arising bicomplex. Then we shall discuss the applications of this theory in topological data analysis, as well as open problems and conjectures.

The talk is based on the recent projects j.w. Anthony Bahri (Rider University, USA), Taras Panov (HSE University, Russia), Jongbaek Song (Pusan National University, Korea) and Donald Stanley (University of Regina, Canada).

Knot Dynamics and Vortex Reconnection

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This talk will discuss how one dimensional vortices in a fluid reconnect to either increase or decrease their topological complexity. We will give examples via photograpy and computer graphics and we shall discuss the problem of finding the least number of reconnections that unknot a given configuration.

The shape operator of real hypersurfaces in $S^6(1)$

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We will present two results concerning real hypersurfaces in the six-dimensional sphere $S^6(1)$. More precisely, we prove that real hypersurfaces with Lie-parallel shape operator A must be totally geodesic hyperspheres. Additionally, we classify real hypersurfaces in nearly Kähler sphere $S^6(1)$ whose Lie derivative of shape operator coincides with its covariant derivative.

Generic bifurcations of tori in 3D integrable Hamiltonian systems

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We study singularities of the Lagrangian fibration given by a completely integrable Hamiltonian system with 2 or 3 degrees of freedom. Our purpose is to study (degenerate) singularities of 'typical' integrable systems.

As a particular case, one can study bifurcations of equilibria in 1-parameter families of integrable systems with 2 degrees of freedom. In this way, Lerman and Umanskii (1981) introduced the so-called parabolic singularity (which is the simplest degenerate singularity of generic integrable systems) and proved that it is structurally stable under (small enough) integrable perturbations of the system.

Another approach is to consider special integrable Hamiltonian systems (called semi-toric), in which the additional first integral generates a Hamiltonian circle action. By combining these two approaches, Duistermaat (1984) and van der Meer (1985) studied the Hamiltonian Hopf bifurcation and its analogue with resonances (that are the simplest generic bifurcations of degenerate equilibria). They proved that such bifurcations appear in generic 1-parameter families of semi-toric integrable systems with 2 degrees of freedom.

A third approach is to study compact periodic orbits instead of bifurcations of relative equilibria (thus the multiplicities of the periodic orbit can be different from 1, allowing a 'twist' of the orbit). By combining this approach with the previous ane (concerning the existence of a circle action), Kalashnikov (1998) described an infinite series of singularities called parabolic orbits with resonances, and proved that they appear in generic 1-parameter families of semi-toric integrable systems with 2 degrees of freedom.

Zung (1997, 2000, 2003, 2006) gave (very mild) sufficient conditions on a compact r-dimensional orbit for the existence of a free system-preserving

Hamiltonian r-torus action near this orbit (hence, the Kalashnikov assumption about the existence of a circle action is not restrictive at all).

We extend the approach by Zung by considering a (so-called "hidden") Hamiltonian torus action near a singular orbit, for real-analytic integrable systems. Such a torus action is generated by first integrals, some of which are multiplied with $\sqrt{-1}$. As we show, such an action is persistent under small real-analytic integrable perturbations of the system (hence, the assumption on the perturbed system to be semi-toric is not restrictive at all). We classified such torus actions up to symplectomorphisms, and proved their rigidity under perturbations.

By combining our approach and the third one (about a twist of the orbit), we extend the bifurcations of equilibria due to Duistermaat and van der Meer (with elliptic resonances) to compact orbits with twists. Using our approach of "hidden" torus actions, we construct two new series of singularities (with hyperbolic resonances and focus-focus resonances) that appear in generic integrable systems with 3 degrees of freedom.

The velocity of one dimension cosmos

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In this work we consider some propreties which is generated by wellknown FRLW metric in the last hundred years . It will be presented geodesic equations in one space dimension. After solving geodesic equations in one space dimension and get the expression for velocity of tuba cosmos, the theoretical results will be compared with astronomical observation to estimate constant k which is parameter of geometric tensor in FRLW metric.

A continuum space is the infinitely great

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Regardless of how modern mathematics develops, it is impossible to calculate exact values (infinite values) with existing calculus because calculus are approximate. For example, for a given line segment or high-dimensional surface, we define it as a manifold that can be infinitely and arbitrarily divided into smaller parts. Here a certain line segment or surface is understood as a set of innumerable infinitesimals and can be calculated by calculus .From the new definition below, it can be seen that this above is not a fact. In this paper an infinitely small quantity is defined as a one-dimensional quantity of finite length without sizes of space, while an infinitely great quantity is reached by the superposition or accumulation of infinitely many finite quantities, by the way of the change in direction. The change in direction indicates that there is a jump from a finite quantity to infinitely many finite quantities (infinitely great). The form of the manifestation of the infinitely great is one quantitative continuum that cannot be operated by any algorithms ,such as the operations of addition, subtraction, multiplication, and division. All parts of space we see is this one quantitative continuum that cannot be talked about anything outside of it and can compresses any quantities outside of it to nothing. The change in direction suggests that the infinitesimal are not parts of infinite great and it does not truely exist (it is only reference quantities that define the infinitely great) due to the infinitely great is only one quantitative continuum where there is only one quantity to exist.Therefore, any line segment or high-dimensional manifold we see in daily life are this one quantitative continuum . As a result from this new definition, the infinite exact value of a circumferential length (π) has been obtained here.

The Jacobi-orthogonality and Osserman tensors

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We introduce a new potential characterization of Riemannian Osserman algebraic curvature tensors. An algebraic curvature tensor is Jacobi-orthogonal if $\mathcal{J}_X Y \perp \mathcal{J}_Y X$ holds for all $X \perp Y$, where $\mathcal{J}$ denotes the Jacobi operator. We prove that any Riemannian Jacobi-orthogonal tensor is Osserman, while all known Riemannian Osserman tensors are Jacobi-orthogonal. We generalize the property of Jacobi-orthogonality to indefinite scalar product spaces. We compare various principles and investigate relations between Osserman, Jacobi-dual, and Jacobi-orthogonal algebraic curvature tensors. We show that every quasi-Clifford tensor is Jacobi-orthogonal. We prove that a Jacobi-diagonalizable Jacobi-orthogonal tensor is Jacobi-dual whenever $\mathcal{J}_X$ has no null eigenvectors for all nonnull X. We show that any algebraic curvature tensor of dimension 3 is Jacobi-orthogonal if and only if it is of constant sectional curvature. We prove that every 4- dimensional Jacobi-diagonalizable algebraic curvature tensor is Jacobi-orthogonal if and only if it is Osserman.

The structure of dendrites and dynamics of continuous maps on them

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By *continuum* we mean a compact connected metric space. A *dendrite* is a locally connected continuum without subsets homeomorphic to a circle. Dendrites appear as Julia sets in complex dynamical systems (see [1]), as ω-limit set of some dynamical systems (see, for example, [2], [3]).

Let X be a dendrite $f : X \to X$ is a continuous map. The structure of dendrites is very complicated (see, for example, [4]). So properties of continuous maps depends on the structure of dendrites (see [5] – [8]). In the report we study the relationship between the structure of dendrites and the dynamics of continuous mappings on dendrites

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References

[1] H.-O. Peitgen, P. Richter. *The Beauty of Fractals*, Springer-Verlag, Heidelberg, (1986).

[2] S.J. Agronsky, J.G. Ceder. *What sets can be ω-limit sets in E^n*, Real Analysis Exchange, **17** (1991-1992), 97-109.

[3] F. Balibrea, J.L. García-Guirao. *Continua with empty interior as ω-limit sets*, Applied General Topology, **6** (2005), 195-205.

[4] K. Kuratovski. *Topology*, vol. 2, Academic Press, New York, (1968).

[5] L.S. Efremova, E.N. Makhrova, *One- dimensional dynamical systems*, Russian Mathematical Surveys 76:5 (2021), 821–881.

[6] E.N. Makhrova. *Monotone maps on dendrites*, Discontinuity, Nonlinearity, and Complexity, **9**:4 (2020), 541-552.

[7] E.N. Makhrova. *The structure of dendrites with the periodic point property*, Rus. Math. (Iz. VUZ), **55**:11 (2011), 33-37.

[8] E.N. Makhrova. *Structure of dendrites admitting an existence of arc horseshoe*, Rus. Math. (Iz. VUZ), **59**:8 (2015), 52-61.

Some curvature properties of quarter-symmetric metric connection

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In this talk, we will discuss the quarter-symmetric metric connection in generalized Riemannian manifolds. We will apply the obtained results to Kähler and cosymplectic (co- Kähler) manifolds, where we will determine tensors that do not depend on the generator of this connection and demonstrate their application in determining specific relations for the Weyl projective curvature tensor and the holomorphically projective curvature tensor.

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Ricci-Bourguignon Almost Solitons with Special Potential on Sasaki-like Almost Contact Complex Riemannian Manifolds

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Almost contact complex Riemannian manifolds, known also as almost contact B-metric manifolds, are equipped with a pair of pseudo-Riemannian metrics that are mutually associated to each other using the tensor structure. Here we consider a special class of these manifolds, those of the Sasaki-like type. They have an interesting geometric interpretation: the complex cone of such a manifold is a holomorphic complex Riemannian manifold (also called a Khler-Norden manifold). The basic metric on the considered manifold is specialized here as a soliton, i.e. has an additional curvature property such that the metric is a self-similar solution of an intrinsic geometric flow. Almost solitons are more general objects than solitons because they use functions rather than constants as coefficients in the defining condition. A β-Ricci-Bourguignon-like almost soliton (β is a real constant) is defined using the pair of metrics. The introduced soliton is a generalization of some well-known (almost) solitons (such as those of Ricci, Schouten, Einstein), which in principle arise from a single metric rather than a pair of metrics. The soliton potential is chosen to be collinear to the Reeb vector field. The resulting manifolds equipped with the introduced almost solitons are characterized geometrically. In the three-dimensional case, an explicit example is constructed and the properties obtained in the theoretical part are confirmed.

Lorentz Transformation and time dilatation

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We consider two inertial frames S and S' and suppose that frame S' moves, for simplicity, in a single direction: the x-direction of frame S with a constant velocity v as measured in frame S.

Using homogenity of space and time we derive modified Lorentz Transformation (LT) between two inertial reference frames without using the second postulate of Einstein, i.e., we does not assume the invariant speed of light (in vacuum) under LT.

Roughly speaking we suppose :

(H) Any clock which is at rest in its frame measures a small increment of time by some factor $s = s(v)$.

For $s = 1$ we get the Galilean transformation of Newtonian physics, which assumes an absolute space and time. We also consider relation between absolute space and Special Relativity Theory, thereafter STR. As a corollary of relativity theory experimentally verified assumption is

(TD): (H) holds with Lorentz factor $1/\gamma$.

It seems here that we need physical explanation for (TD).

We show that (TD) is equivalent with

Postulate 3. The two- way speed is c in any inertial frame.

Note that Postulate 3 is a weaker assumption than Einstein second postulate.

We can derive the corresponding (LT) and explain twin paradox.

We introduce Postulate 3. The two- way speed of light in x' and z'-directions of the frame S' are c, and outline derivation of (LT) in this setting. Note that Postulate 3 is a weaker assumption than Einstein second postulate.

We also consider connection with hyperbolic and Minkowski geometry.

Calculation of quiver representations of finite ring digraphs

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The authors have introduced finite ring digraphs [1] and recently studied the quiver point of view [2]. This is a continuation of this work, containing calculations of quiver representations which correspond to simplest types of these digraphs, in an attempt to introduce cohomological techniques in the topic.

References

[1] A. T. Lipkovski: *Strukturnie grafi kolec: opredelenia i pervie rezuljtati.* Fundamentallnaja i prikladnaja matematika, 2015, tom 20, No 3, s. 181190.

[2] A. T. Lipkovski, J. Matovic: *Quivers associated with finite rings - a cohomological approach.* Filomat 37:25 (2023), 85838589

New types of mappings of generalized Riemannian spaces

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The talk deals with some new types of mappings of generalized Riemannian spaces in the Eisenhart sense. Some relations between corresponding curvature tensors of the generalized Riemannian spaces GR_N and $G\overline{R}_N$ are presented, as well as some invariant geometric objects.

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Minimal models of nilmanifolds and complex structures

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Let G be a real nilpotent Lie group with the tangent Lie algebra $\mathfrak{g}$ and $J : \mathfrak{g} \to \mathfrak{g}$ define a left-invariant almost complex structure on G. The complexification $\mathfrak{g}^*_{\mathbb{C}}$ of its dual space $\mathfrak{g}^*$ splits to

$$(\mathfrak{g}^*)^{\mathbb{C}} = \Lambda^{1,0} \oplus \Lambda^{0,1}, \tag{4}$$

where $\Lambda^{1,0}$ and $\Lambda^{0,1}$ are eigen-spaces of $J^*_{\mathbb{C}} : \mathfrak{g}^* \to \mathfrak{g}^*$ corresponding to $\lambda = i, -i,$. J is integrable if and only if

$$d\Lambda^{1,0} \subset \Lambda^{1,0} \wedge \left(\Lambda^{1,0} \oplus \Lambda^{0,1}\right) = \Lambda^{2,0} \oplus \Lambda^{1,1},$$

where $d : \mathfrak{g}^*_{\mathbb{C}} \to \mathfrak{g}^*_{\mathbb{C}} \wedge \mathfrak{g}^*_{\mathbb{C}}$ is dual to the Lie bracket $[,]_{\mathbb{C}} : \mathfrak{g}_{\mathbb{C}} \wedge \mathfrak{g}_{\mathbb{C}} \to \mathfrak{g}_{\mathbb{C}}$.

We define an increasing sequence of subspaces in the dual space $\mathfrak{g}^*$

$$0 \subset V_1\mathfrak{g}^* \subset V_2\mathfrak{g}^* \subset \cdots \subset V_l\mathfrak{g}^* \subset \cdots \subset V_s\mathfrak{g}^* = \mathfrak{g}^*,$$

where $V_l\mathfrak{g}^* = (\mathfrak{g}^{l+1})^{ann}$ is the annihilator of the ideal $\mathfrak{g}^{l+1}$ of the lower central series We define also the increasing sequence of holomorphic subspaces $V_l^{1,0}, l \geq 1$, of $\Lambda^{1,0}$

$$V_l^{1,0} = (V_l\mathfrak{g}^*)^{\mathbb{C}} \cap \Lambda^{1,0},\ l = 1, \ldots, s.$$

Successively expanding the bases of $V_l^{1,0}$ Salamon [1] constructed a special basis $\omega_1, \ldots, \omega_n$ of the space $\Lambda^{1,0}$ of holomorphic forms

$$d\omega^{l+1} \in I(\omega^1, \ldots \omega^l),\ l = 0, \ldots, n{-}1, \tag{5}$$

where $I(\omega^1, \dots \omega^l)$ is the ideal generated by forms $\omega^1, \dots \omega^l$ [1]. There are complex structures J (called nilpotent [2], [3]) for which the stronger condition of decomposability of d is satisfied

$$d\omega^{l+1} \in \Lambda^2(\omega^1, \dots \omega^l, \bar{\omega}^1, \dots \bar{\omega}^l),\ l = 0, \dots, n-1, \tag{6}$$

The differential algebra generated by $\omega^1, \dots \omega^l$ can be considered as a special minimal model $\mathcal{M}_{\mathfrak{g}}^J$ of a nilmanifold G/Γ, where Γ is a cocompact lattice. The existence of nilpotent complex structure J imposes restrictions on the structure of $\mathfrak{g}$. For example, an estimate for the first Betti number $b_1(\mathfrak{g}) \geq 3$. Recently, interest has grown in nilmanifolds, that admit only nonnilpotent complex structures. The first such example was found by Salamon [1]. In [3] Lie algebras with a one-dimensional center were studied, they also do not admit nilpotent complex structures.

We introduce the notion of a special minimal model $\mathcal{M}_{\mathfrak{g}}^J$, which is constructed from the nilpotent Lie algebra $\mathfrak{g}$ taking into account the complex structure of J. With its help the theorem is proved.

Theorem A real 8-dimensional nilpotent Lie algebra $\mathfrak{g}$ such that $b_1(\mathfrak{g}) = 2$ admits an integrable complex structure if and only if $\mathfrak{g}$ is isomorphic to one and only one Lie algebra from the following five Lie algebras

$$\mathcal{L}(2,4), \mathfrak{n}_1^+(5)_0, \mathfrak{n}_1^+(5)_1, \mathfrak{g}_0, \mathfrak{g}_1.$$

In [1], [3] a short notation of commutation relations for nilpotent Lie algebras is used. With its help we write the relations of

- $\mathcal{L}(2,4) = (0, 0, 12, 13, 23, 14 + 25, 14 - 25, 15 + 24)$;
- $\mathfrak{n}_1^+(5)_t = (0, 0, 12, 13, 23, 14 + 25, 16 + 35 + t \cdot (14 - 25), -26 + 34 + t \cdot (15 - 24)$;
- $\mathfrak{g}_t = (0, 0, 12, 13, 23, 14 + 25, 14 + 3 \cdot 25, 16 + 27 + t \cdot (14 + 25)$.

Note also that the Lie algebras $\mathfrak{n}_1^+(5)_t, t = 0, 1$, are contained in the list [3] of 8- dimensional nilpotent Lie algebras with one-dimensional center and admitting a complex structure.

References

[1] S.M. Salamon, *Complex Structures on Nilpotent Lie Algebras*, J.Pure Appl.Algebra, **157** (2001), 311–333.

[2] L.A. Cordero, M. Fernandez, A. Gray, L. Ugarte, *Compact nilmanifolds with nilpotent complex structures: Dolbeault cohomology*, Trans. Amer. Math. Soc. **352**:2 (2000), 5405–5433.

[3] A. Latorre, L. Ugarte, R. Villacampa, *Complex structures on nilpotent Lie algebras with one-dimensional center*, J. of Algebra, **614** (2024), 271–306

Timelike Surfaces with Parallel Normalized Mean Curvature Vector Field and their Canonical Parameters

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We introduce special isotropic parameters (which we call canonical parameters) for the class of timelike surfaces with parallel normalized mean curvature vector field in the Minkowski 4-space. These parameters allow us to describe this class of surfaces in terms of three geometrically determined functions. We prove a Fundamental existence and uniqueness theorem stating that each timelike surface with parallel normalized mean curvature vector field is determined up to a rigid motion in the Minkowski space by three geometric functions satisfying a system of three partial differential equations. In this way, we minimize the number of functions and the number of equations determining the surface, thus solving the Lund-Regge problem for this class of surfaces.

The talk is based on a joint work with Victoria Bencheva.

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Maslov index and infinitesimal Lagrangian manifolds

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The definition of the Maslov index of a Lagrangian manifold as a class of one-dimensional cohomology on it has given rise to numerous works generalizing the concepts of the Maslov index. In the works of V.I.Arnold [2,3], V.A.Vasiliev[4] and their followers, the theory of Lagrangian bordisms was developed and characteristic classes of Lagrangian submanifolds were constructed on its basis.

There is another approach to describing Maslov classes of Lagrangian submanifolds, presented in the works of V.V.Trofimov and A.T.Fomenko [5] from a categorical point of view.

In addition, there is a series of works by M.V.Karasev and V.P.Maslov [6] in which the canonical operator is constructed for general symplectic manifolds by covering a symplectic manifold with charts diffeomorphic to the simplest symplectic manifold.

Inspired by the works of V.V.Trofimov and A.T.Fomenko, we introduce the concept of so-called infinitesimal Lagrangian manifolds, which, in our opinion, allow us to characterize the characteristic classes of Lagrangian manifolds with maximum completeness and calculate the Maslov index for almost any Lagrangian manifolds.

The Maslov index is constructed as a homological invariant on a Lagrangian submanifold of some symplectic manifold. In the simplest case, the Lagrangian submanifold $\Lambda \subset \mathbb{R}^{2n} \approx \mathbb{R}^n \oplus \mathbb{R}^n$ is a submanifold in the symplectic space $\mathbb{R}^n \oplus \mathbb{R}^n$, the symplectic structure in which is given by the non-degenerate form $\omega = \sum_{i=1}^{n} dx^i \wedge dy^i$, and $\Lambda \subset \mathbb{R}^{2n}$ — is a submanifold, $\dim \Lambda = n$, on which the form ω is trivial. In the general case, the symplectic manifold (W, ω) is considered and a bundle of Lagrangian Grassmannians over a base parametrized by points of a symplectic manifold W.

The question that interests us is the following: when the Maslov index, given on an individual Lagrangian manifold as a one-dimensional cohomology class, is the image of some one-dimensional cohomology class of the total bundle space of Lagrangian Grassmannians. An answer is given for various classes of bundles of Lagrangian Grassmannians.

In particular, homological conditions are found for symplectic manifolds, under which explicit formulas are constructed for calculating the Maslov index of Lagrangian submanifolds.

The infinitesimal approach allows us to use the method of V.V.Trofimov again. Judging by his work (1994) [7], V.V.Trofimov was not satisfied with his generalization of Maslov classes, since with a large holonomy group, Maslov classes are lost. Here, too, we propose to formulate the problem in a categorical language. The object is a pair $h : \Lambda \subset W$, a Lagrangian submanifold Λ in a symplectic manifold W.

It is proposed to replace the symplectic manifold W with a smaller symplectic submanifold $h' : \Lambda \subset W' \subset W$. We get a wider set of characteristic classes, to which the old classes are reduced. Note that when defining characteristic classes, we use only the tangent bundle structure $\mathbb{T}(W)$ to a symplectic manifold W, a natural idea arises to replace the pair $h : \Lambda \subset W$ on the Lagrangian manifold Λ itself, which is equipped with a Lagrangian structure using two vector bundles: tangent bundle of Lagrangian manifold and complex vector bundle with symplectic structure.

Conclusions.

1. Every infinitesimal Lagrangian manifold can be realized as a Lagrangian submanifold of some symplectic manifold whose structure group of the tangent bundle is reduced to the subgroup $\mathbb{O}(n)$. This is enough to describe one-dimensional Maslov classes for an infinitesimal Lagrangian manifold.

2. The symplectic manifold from the previous paragraph can be chosen as the total space of the cotangent bundle of some n-dimensional manifold. It is unclear whether it is possible to embed an infinitesimal Lagrangian manifold into a compact symplectic manifold.

3. The problem of applying infinitesimal Lagrangian manifolds to asymptotic methods in mathematical physics equations is awaiting its solution.

References

[1] A. S. Mishchenko, *Maslov Index on Symplectic Manifolds. With Supplement by A. T. Fomenko "Constructing the Generalized Maslov Class for the Total Space $W = T^*(M)$ of the Cotangent Bundle"*, Mathematical Notes, 112, 697708 (2022).

[2] V. I. Arnol'd, *Lagrange and legendre cobordisms.* I, Functional Analysis and Its Applications, 14, 167177 (1980).

[3] V. I. Arnol'd, *Lagrange and legendre cobordisms.* II, Functional Analysis and Its Applications, 14, 252260 (1980).

[4] A. M. Vassiliev, *Lagrange and Legendre characteristic classes,* 2-d edition, Gordon and Breach Publishers, New York a.o., 1993, 273 pp.

[5] V. V. Trofimov, A. T. Fomenko, *Algebra and geometry of integrable Hamiltonian differential equations, Mat. Prilozh.*[Mathematics and its Applications] Izdatelstvo "Faktorial", Moscow; copublished with Izdatelstvo Udmurtskogo Universiteta, Izhevsk, 1995. 448 pp.

[6] M. V. Karasev, V. P. Maslov, *Pseudodifferential operators and a canonical operator in general symplectic manifolds*, Mathematics of the USSR-Izvestiya, Volume 23, Number 2, 277–305.

[7] V. V. Trofimov, *Generalized Maslov classes on the path space of a symplectic manifold*. Trudy Mat. Inst. Steklov.205(1994), 172199; translation in Proc. Steklov Inst. Math.(1995), no.4, 157179.

Weyl transform of a measure

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(1) Suppose μ is a smooth measure on a smooth hypersurface of positive Gaussian curvature in R^{2n}. $n \geq 2$, then $W(\mu)$, the Weyl transform of μ, is a compact operator, and if $p > n \geq 6$, then $W(\mu)$ belongs to the p-Schatten class.

(2) There exist Schatten class operators with linearly dependent quantum translates.

On deformations preserving dual arc length in dual 3-space

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In this talk, we will discuss the geometrical aspects of dual curves, their properties and characterizations. Based on E. Study's mapping, the set of all oriented lines in Euclidean 3-space is in one to one correspondence with the set of points of dual unit sphere in dual 3-space and a differentiable curve on dual unit sphere represents a ruled surface. We will define a deformation of dual curves preserving dual arc length and consider some geometrical properties under this type of deformation. Consequently, observing the deformations of dual spherical curves we will actually study the deformations of the corresponding ruled surfaces. This is a joint work with Ljubica Velimirović and Svetozar Rančić.

Acknowledgement. This talk is supported by the project IJ2303 of Faculty of Sciences and Mathematics, University of Priština in Kosovska Mitrovica.

On the stability of the identity map of compact space forms

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In this paper, we report on some recent results concerning the stability of the identity map on compact locally conformal almost cosymplectic manifolds of pointwise constant ϕ-holomorphic sectional curvature and compact T-space forms.

On geodesic orbit nilmanifolds

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The talk is devoted to recent results on geodesic orbit Riemannian metrics on nilpotent Lie groups (geodesic orbit nilmanifolds). In particular, we discuss the construction of continuous families of pairwise non-isomorphic connected and simply connected nilpotent Lie groups, every of which admits geodesic orbit metrics. The minimum dimension of such groups is 10.

Killing tensors on symmetric spaces

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I will discuss some known and new results, including a surprising construction showing that the algebra of Killing tensors on a Riemannian symmetric space is not in general generated by Killing vectors. This is a joint work with V. Matveev (Jena, Germany).

Implementation of Postprocessing Procedure of a Rapid Algorithm of Geometric Coding of Digital Images Using CUDA Architecture

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The talk presents some improvements of fast contour recognition algorithm. The main idea of this algorithm is based on calculation of geometric characteristics of a two-dimensional surface in R3, which encodes digital images. To obtain the contours, Minkowski fractal dimension calculation is used. A postprocessing algorithm of contour thinning without loss of quality of entire contour picture is proposed in the paper. An estimate of the speed of the new algorithm is presented in comparison with the well-known implementation of the Canny algorithm using the OpenCV computer vision library and the parallel architecture of CUDA. Examples demonstrating the efficiency of the new algorithm are presented.

This is a joint work with A.Yu Chekunov.

Fischer-Marsden Equation on Paracontact Geometry

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Let (M^{2n+1}, g) be a compact, orientable semi-Riemannian manifold. We denote the set of all unit volume semi-Riemannian metrics on (M^{2n+1}, g) by $\mathcal{M}$. The linearization of the scalar curvature $\mathcal{L}_g(g^*)$ is given by

$$\mathcal{L}_g g^* = -\Delta_g(tr_g g^*) + div(div(g^*)) - g(g^*, S_g),$$

where Δ_g, div, g^* and S_g denotes the negative Laplacian of the semi-Riemannian metric g, divergence operator, symmetric $(0,2)$ tensor field on M and the Ricci tensor, resp. The formal L^2-adjoint $\mathcal{L}_g g^*$ of the linearized scalar curvature operator $\mathcal{L}_g$ is defined by

$$\mathcal{L}_g^*(\lambda) = -(\Delta_g \lambda)g + Hess_g \lambda - \lambda S_g, \tag{7}$$

where $Hess_g \lambda(X, Y) = \nabla_g^2 \lambda(X, Y) = g(\nabla_X D\lambda, Y)$ is the Hessian operator of the smooth function λ on M and D is the gradient operator of g. We refer the equation $\mathcal{L}_g^*(\lambda) = 0$ as Fischer-Marsden equation (FME).

The goal of this study is to investigate the non-trivial solutions of Fischer-Marsden equation on paracontact geometry.

Joint work with İrem Küpeli Erken.

Characterizations of Ricci-Bourguignon almost solitons

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We characterize RB almost solitons on contact metric manifolds and also find some sufficient conditions under which these are isometric to a unit sphere or trivial (Einstein).

Integrability of the sub-Riemannian geodesic flow of the left-invariant metric on the Heisenberg group

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We study two different classes of normal geodesic flows corresponding to the left-invariant sub-Riemannian metric on the $(2n + 1)$-dimensional Heisenberg group. The first class corresponds to the left-invariant distribution, while the second corresponds to the right-invariant one. We show that corresponding Hamiltonian L-L and L-R systems are completely integrable.

Li-Yau sub-gradient estimates and Perelman-type entropy formulas for the heat equation in quaternionic contact geometry

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We are going to present in this talk two sub-gradient estimates for the quaternionic contact (qc) heat equation on a compact qc manifold of dimension $4n+3$, provided some positivity conditions are satisfied. These are qc versions of the prominent Li-Yau gradient estimate in Riemannian geometry. Another goal of the talk is to demonstrate two Perelman-type entropy formulas for the qc heat equation on a compact qc-Einstein manifold of dimension $4n+3$ with non-negative qc scalar curvature (e.g. compact 3-Sasakian manifold), as well as an integral sub-gradient estimate for the positive solutions of the qc heat equation.

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Composition of conformal and projective mappings of generalized Riemannian spaces in Eisenhart's sense preserving certain tensors

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Conformal and projective mappings have important physical interpretations in space-times. These mappings have been thoroughly studied. Their composition between Riemannian spaces that was at the same time harmonic had been initiated by S. E. Stepanov, I. G. Shandra, Seven Classes of Harmonic Diffeomorphisms, Mathematical Notes, 2003, Volume 74, Issue 5, Pages 708-716 and further studied by I. Hinterleitner, Special mappings of equidistant spaces, Ph.D. Thesis, Brno University of Technology, Brno, 2009. Conformal and projective mappings of Riemannian spaces preserving certain tensors were studied by O. Chepurna, Diffeomorphisms of Riemannian spaces with preserved Einstein tensor, Ph.D. thesis, Faculty of Science, Palacky University, Olomouc, 2012. We consider conformal and projective mappings of generalized Riemannian spaces in Eisenhart's sense and find necessary and sufficient conditions for these mappings to preserve curvature, Ricci and Einstein tensors and some of their linear combinations. Particularly, as an additional contribution to related results collected in the Ph.D. thesis by O. Chepurna, we find that the following result holds in the case of Riemannian spaces: if a conformal mapping $f_1 : M \to \hat{M}$ is preserving the Einstein tensor and a projective mapping $f_2 : \hat{M} \to \overline{M}$ is preserving the Einstein tensor then the Yano tensor of concircular curvature is invariant with respect to the composition $f_3 = f_1 \circ f_2 : M \to \overline{M}$.

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Numerical algorithms on Hadamard manifolds

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Numerical procedures for the reckoning of fixed points are analysed from a qualitative point of view in the geometric setting offered by Hadamard manifolds. Convergence, stability and data dependence results are presented, including applications.

The geometrical personalization of human organs 3D models by using the Characteristic Product Features methodology

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Computer-assisted orthopedic surgery (CAOS) involves applying various computer-based methodologies and devices to plan, guide, and perform surgical procedures, thereby improving outcomes throughout the surgical process. This study integrates the Characteristic Product Features (CPFs) methodology developed in-house to improve CAOS. It enables the creation of the human organs' geometrical models by including different relations between Regions of Interest (RGIs) models and specific properties, like functional, materials, and topological. Enhancing existing methodologies in CAOS aims to offer a more comprehensive geometrical description of human organs, leading to the development of more precise and anatomically accurate personalized geometrical models. Creating customized geometry with accurately defined features is expected to enable surgeons to prepare and execute surgical interventions better, consequently improving patient care and recovery prospects. The demonstration of successful geometry adaptation will be shown by prototyping developed models using 3D FDM printing.

This is a joint work with Nikola Vitković, Ljiljana Radović, Jelena Stojković, Aleksandar Miltenović

Basins of attractions of a new iterative method for finding simple zeros

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We present new fourth order iterative method for solving nonlinear equations. New method is obtained by composition of super Halley's and Homeier's three-order methods. Frequently, comparisons of the various schemes are based on the number of iterations required for convergence, number of function evaluations, and/or amount of CPU time. In addition to these criteria, we consider criteria based on basins of attraction. The basin of attraction is a method to visually comprehend how an algorithm behaves as a function of the various initial values. Through many numerical examples it is shown that new method can be competitive to other fourth-order methods.

AMS(MOS) subject classification: 65H05

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The Brylinski beta function of a double layer

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An analogue of Brylinski's knot beta function is defined for a compactly supported (Schwartz) distribution T on d-dimensional Euclidean space. This is a holomorphic function on a right half-plane. If T is a (uniform) double-layer on a compact smooth hypersurface, then the beta function has an analytic continuation to the complex plane as a meromorphic function, and the residues are integrals of invariants of the second fundamental form. The first few residues are computed when $d = 2$ and $d = 3$.

Weak nearly Sasakian and weak nearly cosymplectic manifolds

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Nearly Kähler structure (J, g) was defined by A. Gray (1970) using the condition that the symmetric part of ∇J vanishes. Nearly Sasakian and nearly cosymplectic structures were defined by D. Blair and his collaborators (1976) as odd- dimensional analogs of nearly Kähler structure. Nicola A., Dileo G. and Yudin I. [1] proved that nearly cosymplectic manifolds are Riemannian products of two kinds, and that nearly Sasakian manifolds of dimension greater than 5 are Sasakian. Weak contact manifolds, see [2], i.e., a nonsingular skew-symmetric tensor instead of the complex structure on the contact distribution, allow us to expand the theory of almost contact metric manifolds. Weak nearly Sasakian and weak nearly cosymplectic structures $(\varphi, Q, \xi, \eta, g)$ were defined in [3] similarly to classical case (when $Q = \mathrm{Id}$) using certain conditions for the symmetric part of $\nabla\varphi$.

In this talk, we present theorems generalizing results of [1]: splitting of weak almost cosymplectic manifolds [4], and characterization of Sasakian manifolds in the class of weak contact metric manifolds [5,6].

References

[1] A.D. Nicola, G. Dileo, and I. Yudin, *On nearly Sasakian and nearly cosymplectic manifolds,* Annali di Matematica, 197 (2018), 127–138

[2] V. Rovenski and R. Wolak, *New metric structures on* $\mathfrak{g}$*-foliations*, Indagationes Mathematicae, 33 (2022), 518–532

[3] V. Rovenski, *Weak nearly Sasakian and weak nearly cosymplectic manifolds.* Mathematics 2023, 11(20), 4377

[4] V. Rovenski, *On the splitting of weak nearly cosymplectic manifolds.* Preprint, 16 pp. arXiv:2310.19518, Accepted in DGA

[5] V. Rovenski, *Foliated structure of weak nearly Sasakian manifolds.* Preprint, 10 pp. arXiv:2312.11411, Accepted in AMPA

[6] V. Rovenski, *Characterization of Sasakian manifolds.* Preprint, 14 pp. arXiv:2401.02752, Accepted in AEJM

Willmore-type variational problem for foliated hypersurfaces

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After T. Willmore [4], many authors, including R. Reilly [2] and Z. Guo [1], searched for immersions of a manifold M in a Riemannian manifold, which are critical points of functionals $W = \int_M H^p \,\mathrm{d}V$ or $J = \int_M \|h\|^p \,\mathrm{d}V$. Here, h is the 2nd fundamental form, H the mean curvature and $p > 0$. An interesting problem is the generalization of the Willmore functional to submanifolds with additional structures, such as almost products or foliations.

In the talk, we discuss critical points of the new Willmore-Reilly type functional $WF = \int_M F(\sigma_1^{\mathcal{F}}, \ldots, \sigma_s^{\mathcal{F}}) \,\mathrm{d}V$ for a hypersurface M in $\mathbb{R}^{n+1}$ equipped with an s-dimensional ($s \le n$) foliation $\mathcal{F}$, see [3]. Here, $F \in C^3(\mathbb{R}^n)$ and $\sigma_i^{\mathcal{F}}$ are elementary symmetric functions of the eigenvalues of h restricted on the leaves of $\mathcal{F}$. The 1st and 2nd variations of such functionals are calculated and the conformal invariance of some of them (type Guo) is proven. The corresponding Euler-Lagrange equations for a critical hypersurface with transverse harmonic foliation are found and examples with $s \le 2$ are given. Critical hypersurfaces of revolution in $\mathbb{R}^{n+1}$ are presented and shown to be a local minimum for special variations of immersion.

References

[1] Z. Guo, *Generalized Willmore functionals and related variational problems.* Diff. Geom. Appl., 25 (2007), 543–551.

[2] R.C. Reilly, *Variational properties of functions of the mean curvatures for hypersurfaces in space forms.* J. Differ. Geom. 8, 465–477 (1973).

[3] V. Rovenski, $\widehat{\rm W}$illmore-type variational problem for foliated hypersurfaces, preprint, 2024, 14 pages. arXiv.2402.17565.

[4] T.J. Willmore, *Note on embedded surfaces.* An. Stiint. Univ. "AI. I. Cuza" Iasi, Sect. Ia Mat. 11. (1965), 493–496.

On the Index Problem On Manifolds with Group Actions. Contributions of Fixed Points

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Given an action of a discrete group G on a smooth closed manifold X, we consider a representation of G by shift operators $T_g : u(x) \mapsto u(g^{-1}x)$ in function spaces on X and a class of matrix operators with shifts of the form

$$D = \sum_{g \in G} D_g T_g : C^\infty(X, \mathbb{C}^N) \longrightarrow C^\infty(X, \mathbb{C}^N),$$

where the coefficients D_g are matrix (pseudo)differential operators on X. If appropriate ellipticity conditions are satisfied, D is a Fredholm operator and the problem arises of calculating its index in terms of the principal symbol $\sigma(D)$ of this operator (see, for example, [1] and the cited literature). It should be noted that the principal symbol is an element of an essentially non-commutative algebra — the crossed product of the algebra of matrix functions on the cospheric bundle of the manifold and the group G acting on the indicated algebra by automorphisms.

In [2], the index was calculated in the case of the group $G = \mathbb{Z}$ and it was shown that the index is equal to the pairing of the class in the K-theory of the crossed product defined by the principal symbol $\sigma(D)$ and the equivariant Todd class of the manifold in periodic cyclic cohomology of the crossed product.

We define the Todd class for groups of the form $G = \mathbb{Z} \oplus F$, where F is a finite group, and prove the corresponding index formula. The main difficulties that had to be overcome: 1) the action of the group, generally speaking, is not assumed to be isometric; 2) it was necessary to describe the contributions of the fixed points of the group action to the index formula.

The results are obtained in a joint work with Haydar Abbas (RUDN University).

References

[1] A. Antonevich, A. Lebedev, *Functional-differential equations. I. C^*-theory.* Pitman Monographs and Surveys in Pure and Applied Mathematics, 70. — Harlow: Longman Scientific& Technical, 1994.

[2] A. Savin, B. Sternin, *Index of elliptic operators for diffeomorphisms of manifolds* J. Noncommut. Geom.,**83**:3 (2014), 695-734.

Geometric Properties of the Inertia Tensor of a Rigid Body

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The report will explore the connection between the following mechanical problems:

- the problem of the rotation of a rigid body around a fixed point (i.e., a spinning top) in a 3-dimensional (pseudo-)Euclidean space,
- the problem of the motion of a rigid body (referred to as a "plate") in a 2-dimensional space of constant curvature, specifically, on a 2-dimensional sphere, a Euclidean plane, or a Lobachevsky plane.

The inertia tensor of the rigid body from mechanics will be examined. We will describe its connection with the kinetic energy of the rigid body and the inertia tensor on the Lie algebras $so(3)$ and $so(2,1)$ in terms of a natural isomorphism between this Lie algebra and the ambient (pseudo-)Euclidean space. We will compute the inertia tensor of any single-point body in terms of the (pseudo-)Euclidean metric of the ambient space. As a consequence, firstly, it follows that the value of the inertia tensor (as a quadratic form) on any time-like vector is non-negative. In particular, the inertia tensor cannot have a signature of $(-,-,-)$. Secondly, for any "plate" on the Lobachevsky plane (lying within the light cone), the inertia tensor is positive definite. Thirdly, we will provide specific examples of two-point bodies lying outside the light cone, whose inertia tensor can have any signature other than $(-,-,-)$.

Invariants for geometric mappings

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In this research, we consider transformations of symmetric affine connections and invariants caused by them. Our results are classified in three sections. The first section is Introduction, where we recalled the basic definitions and motivations necessary for the research. In the second section, we considered transformations of affine connections generated with respect to metric tensors. In this section, we obtained the invariant geometrical objects with respect to the transformations of affine connections and curvature tensors of symmetric affine connection spaces with metrics defined on them. In the third section, we presented an example of application of results presented in the second section.

Key words: Riemannian space, sense, invariant

Math. Subj. Classification: 53A55, 53B20

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On some generalizations of Kahlerian spaces

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In this talk, we present results on generalized Kahlerian spaces [1], generalized Kahlerian spaces of the first kind [2], and generalized Kahlerian spaces of the second kind [3]. In all three cases, invariant geometric objects of holomorphic projective mappings of generalized Kahlerian spaces, of the Weyl tensor type, are considered.

The author acknowledge the grant of the Ministry of Education, Science and Technological Development of Serbia 451-03-65/2024-03/200124 for carrying out this research.

References

[1] M. S. Stankovic, S. M. Mincic, Lj. S. Velimirovic, *On equitorsion holomorphically projective mappings of generalised Kahlerian spaces*, Czechoslovak Mathematical Journal, vol.54 (129) , No. 3 (2004) 701-715.

[2] M. S. Stankovic, M. Lj. Zlatanovic, Lj. S. Velimirovic, *Equitorsion holomorphically projective mappings of generalized Kahlerian space of the first kind*, Czechoslovak Mathematical Journal, vol.Vol. 60, No. 3 (2010) 635-653.

[3] M. S. Stankovic, M. Lj. Zlatanovic, L. S. Velimirovic, *Equitorsion holomorphically projective mappings of generalized Kahlerian space of the second kind*, International Electronic Journal of Geometry, (M50) vol.Vol. 3, No. 2 (2010) 26-39.

New vanishing theorem of conformal Killing forms

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In this talk we generalize our result (see [1]) and results of Kashiwada and Kora (see [2] and [3]) in the form of the following theorem.

Theorem. Let (M, g) be an n-dimensional closed connected Riemannian manifold with non-positive curvature operator of the second kind. Then either it is a compact Euclidean space form, or it does not admit conformal Killing p-forms for all $p = 1, 2, \cdots, n1$.

References

[1] S.E. Stepanov, I.I. Tsyganok, *Theorems of existence and non-existence of conformal Killing forms*, Russian Math. (Iz. VUZ), 58:10 (2014), 4651

[2] T. Kashiwada, *On conformal Killing tensor*, Natural Science Report, Ochanomozu University, 19:2 (1968), 67-74.

[3] M. Kora, *On conformal Killing forms and the proper space of* Δ *for p-forms*, Mathematical Journal of Okayama University, 22:2 (1980), 195-204.

Geometrical eigenproblem of various types higher order tensors in Riemannian space

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The geometrical eigenproblem (G-eigenproblem) is validated as an extension of the classical eigenproblem of a linear operator within Euclidean vector space. It is homogeneous and invariant, making it efficient and advantageous for tensors describing real phenomena, regardless of the representation, while determining their eigenspaces. The G-eigenproblem in Riemannian space will be explored not only for covariant tensors but also for mixed and contravariant ones. Achievements will be discussed and related to higher-order unit tensors.

3-Triangulation of polyhedra and their connection graphs

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Here we investigate the properties of 3-triangulation of polyhedra, when possible. Namely, it is known that 3- triangulation of convex polyhedra is always possible, but this is not the case with all non-convex ones. This is the reason to consider the decomposition of non-convex polyhedra into convex pieces if possible. After that, we introduce the connection graph for the 3-triangulable polyhedron in such a way that these pieces are represented by the nodes of the graph.

First, our attention will be focused to the minimal number of tetrahedra necessary to 3-triangulate toroids, a special class of non-convex polyhedra. Polyhedra that are topologically equivalent to ball with p handles ($p \in \mathbb{N}$) are called p-toroids, and more generally, independently on p, we call them toroids.

As another application of connection graphs, we shall also consider those corresponding to convex polyhedra, especially to conic triangulation of them.

Gromov-Hausdorff Distance and Geometric Optimization Problems

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We will discuss how the Gromov-Hausdorff distance arises in various combinatorial and geometric optimization problems. First, we recall our results (obtained jointly with Professor Alexander O. Ivanov) on (1) Borsuk's problem about the smallest number of parts into which a bounded subset of Euclidean space can be cut so that all parts have a smaller diameter than the original subset; (2) finding the smallest number of colors necessary to correctly color the vertices of the graph; (3) calculating the smallest number of cliques whose vertices cover the set of vertices of the graph; (4) calculating the weights of the edges of the minimum spanning tree in the weighted graph. After this, we will show how to express through the Gromov- Hausdorff distance the l_1-dimension, i.e., the smallest dimension of a vector space with Manhattan (l_1-) norm into which a given finite metric space can be isometrically embedded.

Size in contact geometry

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In this talk, I will cover the basics of contact manifolds - defining them and highlighting their main properties. I will discuss the concept of contact non-squeezing, a handy tool for understanding size when there is no natural measure or metric. Lastly, I will share an interesting finding: while contact geometry does not capture size of a ball on a small scale, it surprisingly does so on a larger scale in certain cases.

Extremal Kähler metrics and the moment map

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An extremal Kähler metric is a canonical Kähler metric, introduced by E. Calabi, which is somewhat more general than a constant scalar curvature Kähler metric. The existence of such a metric is an ongoing research subject and expected to be equivalent to some form of geometric stability of the underlying polarized complex manifold $(M, J, [\omega])$ –the *Yau-Tian-Donaldson conjecture*. Thus it is no surprise that there is a moment map, the scalar curvature (A. Fujiki, S. Donaldson), and the problem can be described as an infinite dimensional version of the familiar finite dimensional G.I.T.

In this talk I will describe how the moment map can be used to describe the local deformation problem of constant scalar curvature and extremal metrics. Essentially, the local picture can be reduced to finite dimensional G.I.T. In particular, we can construct a course moduli space of extremal Kähler metrics with a fixed polarization $[\omega] \in H^2(M, \mathbb{R})$, which is an Hausdorff complex analytic space.

Conformal transformations of curvature pseudotensors

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In this talk we study curvature pseudotensors. They are tensors constructed from the curvature tensor and other geometric quantities. However, they are not true tensors since they don't transform like tensors under arbitrary coordinate transformations. Instead, they transform according to certain rules under specific transformations, such as conformal transformations. We exam ET conformal transformations at GR_N.

Curvature based shape consideration

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The study of deformations dates from the ancient times and stems from purely practical problems. Deformations are in close connection with thin elastic shell and has a huge application from the mechanical point of view. In biology, the notion of deformation has also found his place. In geometry, the problem of deformations is covered by so-called the **surface bending theory**.

The surface bending theory considers the bending of surfaces, ie. the isometrical deformations, as well as, the infinitesimal bending of surfaces and presents one of the main parts of **global differential geometry**. In this talk, we want to introduce the manuscript of a book "Shape and Energies of Geometric Objects" whose authors are L. Kauffman, M. Najdanovic, S. Rancic and me. We consider shape of deformable objects at small, infinitesimal deformations. It is determined by their geometric curvature as well as curvature-based functionals which are said to be the energies of Geometric objects. Geometric objects that are discussed are curves surfaces and knots. To study energies and shape of curves and knots we use Willmore energy, total curvature, total torsion, total normalcy and Mbius energy. We expect that our investigations of energy and Willmore energy will eventually shed light on such phenomena.

Structural Shape Analysis

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Contemporary architectural and structural designs embrace irregular or freeform surfaces for their aesthetic appeal. The structural shape has important role in various aspects of design, analysis and construction. We have employed mathematical analysis of curvature and curvature based functions in order to understand surface characteristics. In civil engineering, adoption of the optimal structural shape is vital to guarantee structures fulfill essential performance criteria such as strength, stability, and functionality during the service life.

Keywords: structural shape, curvature, civil engineering

A generalization of a result of Minakshisundaram and Pleijel

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Minakshisundaram and Pleijel gave an asymptotic formula for the sum of squares of the pointwise values of the eigenfunctions of the Laplace-Beltrami operator on a compact Riemannian manifold, with eigenvalues less than a fixed number. Here, a generalization is given, where the pointwise values are replaced by the Fourier coefficients of a smooth measure supported on a compact submanifold.

New invariants for conformal mappings of Riemannian spaces

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In this research, we are aimed to complete the study of invariants for conformal mappings of Riemannian spaces. Until now, the Weyl conformal tensor is the only invariant for this mapping which has been used. We obtained the invariant for conformal mapping f which is analogous to the Thomas projective parameter. After that, we obtained two invariants for conformal mappings with respect to transformation of Riemann-Christoffell curvature tensor of a Riemannian space. One of these two invariants is the Weyl conformal curvature tensor, but the second one is independent of this known invariant.

Key words: conformal mapping, basic invariants, weyl conformal tensor

Math. Subj. Classification: 53A55, 53B20

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On Yamada polynomial of spatial graphs and Jones polynomial of related links

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We will discuss invariants of spatial embeddings of graphs into a 3-sphere. Since any simple cycle of a graph is embedding as a knot, the theory of spatial graphs can be considered as a generalization of the knot theory. Two spatial graphs are said to be equivalent if there is an ambient isotopy of the 3-sphere which transforms one spatial graph to another. As well as knots and links, spatial graphs can be studied from their diagrams and Seifert surfaces. It was shown in [1] that an invariant link can be associated to an embedding a planar three-valent graph with at most six edges. The Yamada polynomial introduced in [2] is the most useful invariant of spatial graphs. A relation between the normalized Yamada polynomial of a spatial theta-graph and the Jones polynomial of the associated link was described in [3]. Let K4 be the complete graph with four vertices. We will present a relation between the normalized Yamada polynomials of a spatial K4-graph and its spatial subgraphs with the Jones polynomial of the associated link, see [4] for details.

References

[1] L.H. Kauffman, J. Simons, K. Wolcott, P. Zhao, *Invariants of theta-curves and other graphs in 3-space.* Topology and Its Applications, 49 (1993), 193-216.

[2] S. Yamada, *An invariant of spatial graphs*, Graph Theory, 13 (1989), 537-531.

[3] Y. Huh, *Yamada polynomial and associated link of theta-curves.* Discrete Mathematics, 347:1 (2024), paper 113684.

[4] O. Oshmarina, A.Vesnin, *Polynomials of complete spatial graphs and Jones polynomial of related links.* 2024, 28 pp. Preprint arXiv:2404.12264.

Geodesics in Gromov-Hausdorff class

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The talk is dedicated to the proper class of all metric spaces considered up to isometry, equipped with the Gromov–Hausdorff distance. More precisely, we discuss the properties of geodesics in this Gromov–Hausdorff class.

The Gromov–Hausdorff distance measures the degree of the difference between two metric spaces. This distance was introduced by Gromov in 1981 and was defined as the smallest Hausdorff distance between isometric images of the considered spaces. Later, an equivalent definition of this distance was given using correspondences.

In Ivanov–Nikolaeva–Tuzhilin work an optimal correspondence between finite metric spaces was used to construct a geodesic between arbitrary compact metric spaces. Later on, the existence of optimal correspondence between compact metric spaces was proved, and as a consequence, a geodesic between these spaces generated by the optimal correspondence was constructed. Such geodesics are called *linear* ones. However, it is still unknown whether any pair of metric spaces at a finite distance from each other can be connected by some geodesic.

The example of two metric spaces without linear geodesic between them is trivial, e.g. [0,1] and (0,1). Ghanaat found an example of two complete non- isometric metric spaces on zero distance, thus, they are not connected by linear geodesic. Hansen and the author found independently similar examples on non- zero distance. However, in the both examples, one can choose another metric spaces on zero distance from the initial ones, such that the latter spaces are connected by a linear geodesic. Later, the author found an example of two complete metric spaces such that no linear geodesics connect them and there are no metric spaces on zero distance. However, there exists a geodesic that are not linear. In the talk another method to construct a geodesic will be presented. It is called Hausdorff realization. In the case two metric spaces have linear geodesic, then one can construct geodesic by means of Hausdorff realization. Nevertheless, there is an example of two metric spaces with no Hausdorff realization (Hansen). We will construct a geodesic in this case as well.

Einstein Foliations

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A Riemannian manifold is said to be *Einstein* whenever its Ricci curvature is constant. A foliation of a Riemannian manifold is *Einstein* whenever all its leaves are Einstein when equipped with induced Riemannian structures. Using Gauss and Codazzi equations, we shall provide conditions necessary and/or sufficient for this property and discuss the problem of existence of foliations of this sort. To simplify calculations and the formulas, we shall focus on the case when the ambient space is locally symmetric.

On statistics which are almost sufficient from the viewpoint of the Fisher metrics

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Given a statistical model, a statistic on the model is sufficient if the Fisher metric of the induced model coincides with the original Fisher metric, according to the definition by Ay-Jost-Lê-Schwachhöfer. We introduce and study its quantitative version: for $0 < \delta \leq 1$, we call a statistic δ-almost sufficient if $\delta^2 \mathfrak{g}(v, v) \leq \mathfrak{g}'(v, v)$ for every tangent vector v of the parameter space, where $\mathfrak{g}$ and $\mathfrak{g}'$ are the Fisher metric of the original and the induced model, respectively. By the monotonicity theorem, the Fisher metric of the induced model for such a statistic is bi-Lipschitz equivalent to the original one, which means that the information loss of the statistic is bounded in a quantitative way. We characterize such statistics in terms of the conditional probability or by the existence of a certain decomposition of the density function in a way similar to the characterizations of sufficient statistics due to Ay-Jost- Lê-Schwachhöfer and Fisher-Neyman.

Legendrian dual surfaces, lying in the 3-dimensional de Sitter space, of a spacelike curve in the 3-dimensional lightcone

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Legendrian dualities and the duality theorem were given in [1] for the pseudo-spheres in Lorentz-Minkowski space. Later, these Legendrian dualities and the duality theorem were extended in [2] for one-parameter families of pseudo-spheres in Lorentz-Minkowski space. In terms of two extended Legendrian dualities and the extended duality theorem in [2] and new extended Legendrian dualities in [3], an extended duality theorem was given in [3] and as an application of this theorem, Legendrian dual surfaces of a spacelike curve in the 3-dimensional lightcone were constructed in [3].

In this talk which is based on the joint work with Kentaro Saji given in [3], after constructing the Legendrian dual surface, lying in the 3-dimensional de Sitter space, of a spacelike curve in the 3-dimensional lightcone, the singularities of this surface are given.

Acknowledgements

A part of [3] was conducted by the fruitful discussions of the speaker and Kentaro Saji during the speaker's participations to some scientific activities through the projects numbered 23429 and 33279 which were supported by the Scientific Research Projects Coordination Unit of Istanbul University. Moreover, a part of [3] was conducted during the speaker's visit to Mathematics Department of Kobe University in 2014. Furthermore, Kentaro Saji was partially supported by JSPS KAKENHI Grant Number 18K03301.

References

[1] S. Izumiya, *Legendrian dualities and spacelike hypersurfaces in the lightcone*, Moscow Mathematical Journal, **9**(2) (2009), 325-357.

[2] S. Izumiya, H. Yıldırım, *Extensions of the mandala of Legendrian dualities for pseudo-spheres in Lorentz-Minkowski space*, Topology and its Applications, **159**(2) (2012), 509-518.

[3] K. Saji, H. Yıldırım, *Legendrian dual surfaces of a spacelike curve in the 3-dimensional lightcone*, Journal of Geometry and Physics, **179** (2022) 104593, 1-21.

Semisimple algebraic Nijenhuis operators on small dimentional Lie algebras

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We consider a problem related to the Nijenhuis geometry.

For arbitrary operator field L the formula

$$\mathcal{N}_A(\xi,\eta) = A[[A\xi,\eta]] + A[[\xi,A\eta]] - [[A\xi,A\eta]] - A^2[[\xi,\eta]]$$

defines $(1,2)$ type tensor field called *Nijenhuis torsion*. Here $[[\,,\,]]$ stands for standard commutator of vector fields. An operator field L is called *Nijenhuis operator* if its Nijenhuis torsion identically vanishes.

A natural class of such operators are left-invariant Nijenhuis operators A on a Lie group G.This means that A is invariant under the left action of the group on itself. In this case the tensor field A is completely defined by a linear operator $L: g \to g$, where g is the Lie algebra of G. Such operator L satisfies the identity

$$L[L\xi,\eta] + L[\xi,L\eta] - [L\xi,L\eta] - L^2[\xi,\eta] = 0,$$

where $[\,,]$ is the Lie algebra commutator. In this case the operator is called an *algebraic Nijenhuis operator*. Such operators appear in a study of completely integrable systems (both classic and quantum) and in many questions in geometry and mathematical physics. But we still have few examples of such operators. Our goal is to classify all algebraic Nijenhuis operators for small dimensional Lie algebras.

There is a theorem due to Y. Kosmann-Schwarzbach, F. Magri providing a characteristic property of such operators. This theorem says that real or complex Lie algebra g admits semisimple algebraic Nijenhuis operator if and only if there exists a basis $\eta_1,\dots,\eta_n$ such that

$$[\eta_i,\eta_j] \in \langle \eta_i,\eta_j \rangle.$$

In other words any two vectors η_i, η_j generate a two-dimensional Lie subalgebra.

Applying this theorem we classified real and complex Lie algebras admitting algebraic Nijenhuis operators in dimensions three and four. For each such Lie algebra we describe all bases satisfying the above condition. An equally interesting question is to describe all algebraic Nijenhuis operators which are not equivalent by an automorphism of the Lie algebra. This problem is not solved yet, but we obtained the answer for some Lie algebras.

(Non) existence of Lagrangians in hyperkähler manifolds

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The purpose of the talk is to convey two phenomena, the existence and non-existence of Lagrangian submanifolds inside hyperkähler manifolds with holomorphic $\mathbb{C}^*$-actions. The first ones arise as the minima of the moment map of the corresponding S^1-actions, whereas the non-existence of the latter ones comes from the vanishing of a certain Floer-theoretic invariant called symplectic cohomology. We will discuss minimal resolutions of A_n-singularities as a basic example.

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