Optimization of Variable-length Code for Data Compression of memoryless Laplacian source

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Abstract

In this paper we present the efficient technique for compression and coding of memoryless Laplacian sources, which uses variable-length code (VLC). That technique is based on combination of two companding quantizers in the first case, and three companding quantizers in the second case. These quantizers have disjoint support regions, different number of representation levels and different compressor functions. The closed-form expressions are obtained for the distortion, average bit rate and signal to quantization noise ratio ($SQNR$). Presented numerical results point out the effects of rate-distortion (R-D) optimization on the system performances. Since our model assumes the general case of Laplacian distribution, it has a wide applications like coding of speech and images. It is shown that the difference of $SQNR$ of our model and classical

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companding quantizer based model is 2.8dB for two quantizers and 4.2dB in three quantizers model. We have also made a comparison between our model, combination of the optimal uniform quantizer and Huffmann lossless coder and combination of optimal companding quantizer and simple lossless coder [16].

Keywords: variable-length code; memoryless Laplacian source; multi-resolution scalar quantizer

1 Introduction

Quantizers play an important role in the theory and practice of modern day signal processing. They are applied for the purpose of storage and transmission of continual signals. All data-compression schemes assume a digital source of information with known statistical properties as an input. The output of the source is a set of symbols with a given probability of occurrence. Compression is achieved by assigning shorter codewords to the more frequent symbols and longer codewords to the less frequent ones. The compressed output is simply the concatenation of such codewords. This is an important application of variable-length codes.

Many sources that we deal with, have distribution which is quite peaked at zero. For example, speech consists mainly of silence. Therefore, samples of speech will be zero or close to zero with high probability. On the other hand, image pixels do not have any attraction to small values. But there is high degree of correlation among pixels. Therefore, a large number of the pixel-to-pixel differences will have values close to zero. In these situations, Laplacian distribution provide a close match to data. Memoryless Laplacian source is commonly used and important in many areas of telecommunications and computer science.

An efficient algorithm for the design of the optimal quantizer for the source with known distribution was developed by Lloyd and Max [1]. However, this method is very time consuming for the large number of quantization levels. One solution which overcomes these difficulties is the companding model [2]-[3]. Quantizers based on the companding model have simple realization structure and performances close to the optimal ones. Its simplicity, parameters, and many characteristics can be described in closed form relations. Examples are: speech signal, images, video signal, etc. The design of such quantizers is also more efficient than Lloyd-Max's algorithm since it does not require the iterative method. This difference is very notable for some commonly used sources including the Laplacian source.

For the purpose of transmission, processing and storing that signals, simple and fast compression algorithms are desirable. One solution is given in [4] where a uniform quantizer is considered. In paper [5] lossless compression algorithm provided only the additional compression of the digitized signal (PCM), but without providing a quality improvement. In [6] forward adaptive technique is given for Lloyd-Max's algorithm implementation in speech coding algorithm. Fixed-rate scalar quantizers for Laplacian source have already been the topic of earlier research [7]-[8]. The well-known efficient algorithm for lossless coding of the information sources with known symbol probability, is Huffman algorithm [9]-[10]. It requires a very complex realization structure and also is time consuming. Hardware implementations of popular compression algorithms such as the Huffman coding [11], Lempel-Ziv coding [12], binary arithmetic coding [13], and the Rice algorithm [14] have been reported in the literature. A 12-bit A/D with a simplified Huffman encoder is presented in [15]. Compression algorithm for Laplacian source, consisting of an optimal bounded companding quantizer and simple lossless coder is given in

[16]. Multi-resolution scalar quantizers are described in [17].

In this paper we give the simple solution based on two non-uniform companding quantizers, in the first case, and three non-uniform companding quantizers, in the second case. For a fixed value of average bitrate R, we provide the optimization of region bounds as well as the number of quantization levels of each companding quantizer.

We compare our model to the combination of the optimal uniform quantizer and Huffmann lossless coder. It is shown that our model possesses better results, with much simple and more efficient realization structure. Comparation is also made with the combination of optimal companding quantizer and simple lossless coder [16]. Model in paper [16] is performing coding and decoding the groups of three samples and transmitting the side information about number of bits (bitrates $R-1$ or R) used for the coding. In our paper coding and decoding is performed sample by sample and transmitted side information is used for determining which quantizer corresponds to which sample. Advantages of our model are better results and higher flexibility in quantizer designing (changeability of number of quantizers and their bitrates), with slightly more complex realization structure.

We also deal with an application of our model in speech signal coding. It is used for compression of the sample speech signal and the obtained experimental results are compared with theory.

This paper is organized as follows. Section 2 recalls some basic theories of quantizers and companding model. In Section 3 we give a description of the variable-length code for data compression of memoryless Laplacian source, which consists of two and tree companding quantizers. Section 4 contains some numerical examples. We also performed an optimization of the quantizer distortion for prescribed value of average bit rate. In Section 5, for the purpose of testing, we considered the adaptive variant of our three quantizer VLC model which is tested on the sample speech signal. Section 6 concludes the paper by summarizing the key features of the coder design and its applications.

2 Scalar quantizers and companding technique

Assume that an input signal is characterized by continuous random variable X with probability density function (PDF) $p(x)$. The first approximation to the long-time-averaged PDF of amplitudes is provided by a two-sided exponential or Laplacian model. Waveforms are sometimes represented in terms of adjacent-sample differences. The PDF of the difference signal for an image waveform follows the Laplacian function [2,p33]. Laplacian source can be also used for modelling of the speech signal [21,p384]. In the rest of the paper we assume that information source is Laplacian source with memoryless property and zero mean value. The PDF of that source is given by:

$$p(x) = \frac{1}{\sqrt{2\sigma^2}}\ e^{-\frac{|x|\sqrt{2}}{\sigma}}, \tag{1}$$

where x is zero-mean statistically independent Laplacian random variable of variance σ^2.

The sources with exponential and Laplacian PDF are commonly encountered and the methods for designing quantizers for these sources are very similar. Without loss of generality we can suppose that $\sigma = 1$ and expression (1) becomes:

$$p(x) = \frac{1}{\sqrt{2}}\ e^{-\sqrt{2}|x|}. \tag{2}$$

An N-point fixed rate scalar quantizer is characterized by the set of real numbers

$t_1, t_2, \ldots, t_N$, called *decision thresholds*, which satisfy

$$-\infty = t_0 < t_1 < \cdots < t_{N-1} < t_N = +\infty, \tag{3}$$

and set $y_1, y_2, \ldots, y_N$, called *representation levels*, which satisfy

$$y_j \in \alpha_j = (t_{j-1}, t_j], \quad \text{for } j = 1, \ldots, N. \tag{4}$$

Sets $\alpha_1, \alpha_2, \ldots, \alpha_N$ form the partition of the set of real numbers $\mathbb{R}$ and are called *quantization cells*. The quantizer is defined as many-to-one mapping $Q : \mathbb{R} \to \mathbb{R}$, $Q(x) = y_j$ where $x \in \alpha_j$. In practice, input signal value x is discretized (quantized) to the value y_j. Cells $\alpha_2, \alpha_3, \ldots, \alpha_{N-1}$ are *inner cells* (or *granular cells*) while α_1 and α_N are *outer cells* (or *overload cells*). In such way, cells $\alpha_2, \alpha_3, \ldots, \alpha_{N-1}$ form granular while cells α_1 and α_N form an overload region. Since variable-rate and scalar quantizers are the only types of quantizers considered in the paper, we just briefly recall their properties.

The quality of the quantizer is measured by distortion of resulting reproduction in comparison to the original one. Mostly used measure of distortion is mean-squared error. It is defined by:

$$D(Q) = E(X - Q(X))^2 = \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} (x - y_i)^2 p(x) dx. \tag{5}$$

The N-point quantizer Q is *optimal* for the source X if there is no other N-point quantizer Q_1 such that $D(Q_1) < D(Q)$. We also define granular distortion $D_g(Q)$ and overload $D_{ol}(Q)$ distortion by:

$$D_g(Q) = \sum_{j=2}^{N-1} \int_{t_{j-1}}^{t_j} (x - y_j)^2 p(x) dx, \tag{6}$$

$$D_{ol}(Q) = \int_{-\infty}^{t_1} (x - y_1)^2 p(x) dx + \int_{t_{N-1}}^{+\infty} (x - y_N)^2 p(x) dx. \tag{7}$$

Obviously follows that $D(Q) = D_g(Q) + D_{ol}(Q)$.

Considerable amount of work has been focused on the design of optimal quantizers for compression sources in image, speech, and other applications. Denote by D_N^* the distortion of an optimal N-point quantizer. As it was discovered by Panter and Dite [11], for large N holds $D_N^* \cong c_\infty/N$. Here c_∞ is the Panter-Dite constant

$$c_\infty = \frac{1}{12}\left(\int_{-\infty}^{+\infty} p^{1/3}(x)dx\right)^3. \tag{8}$$

The general method for the design of an optimal N-point quantizer for the given source X is Lloyd-Max algorithm [1], [2], [20]. Due to the computational complexity of this method, it is not suitable for the design of optimal quantizers with more than 128 levels. Hence, other methods for the construction of nearly optimal quantizers for large number of quantization levels are developed. One of the commonly used techniques for this purpose is the companding technique [12]. It forms the core of the ITU-T G.711 PCM standard, recommended for coding speech signals. Companding technique consists of the following steps:

1) Compress the input signal x by applying the compressor function $c(x)$.

2) Apply the uniform quantizer Q_u on the compressed signal.

3) Expand the quantized version of the compressed signal using an inverse compressor function $c^{-1}(x)$.

The corresponding non-uniform quantizer consisting of a compressor, a uniform quantizer, and an expander in cascade is called *companding quantizer* (*compandor*). Hence, the companding quantizer can be represented as $Q(x) = c^{-1}(Q_u(c(x)))$, where $Q_u(x)$ is uniform quantizer in the interval $[-1, 1]$. Denote by $t_{u,i}$ and $y_{u,i}$ decision thresholds and representation levels of the uniform quantizer $Q_u(x)$. Corresponding values t_i and y_i of the companding quantizer $Q(x)$ can be determined as the solutions of the following equations:

$$c(t_i) = t_{u,i} = -1 + \frac{2i}{N}, \quad c(y_i) = y_{u,i} = -1 + \frac{2i-1}{N}. \tag{9}$$

There are several ways how to choose the compressor function $c(x)$ for compression law. Originally, in [12] and also in [8], compressor function $c_0 : \mathbb{R} \to [-1, 1]$ is defined as

$$c_0(x) = \frac{\int\limits_{-\infty}^{x} p^{1/3}(x)dx}{\int\limits_{-\infty}^{+\infty} p^{1/3}(x)dx}. \tag{10}$$

In this paper, we use the similar definition of the compressor function which will be described in the following section.

3 Description and construction of VLC coder and decoder

In this section we describe our model consisting of two and three companding quantizers, in the first and the second case respectively. Optimization of the bounds of support regions and numbers of representation levels, is performed for a fixed average bitrate R.

3.1 Two companding quantizers VLC model

The coder consists of two companding quantizers with different number of representation levels and different compressor functions. First quantizer Q_1 is applied on the inner segment $I = [-t_1, t_1]$, while the second quantizer Q_2 is applied on the outer segment $O = (-\infty, -t_1] \cup [t_1, +\infty)$. Value t_1 is called the threshold value. We denote by $N_i = 2^{k_i}$ the number of the quantization levels of quantizer Q_i, where k_i

is number of bits per sample and $i = 1, 2$. Let $c_1 : I \to [-1, 1]$ and $c_2 : O \to [-1, 1]$ be the corresponding compressor functions. An optimal compressor function c_1 is given by the following expression (Judell and Scharf, [18]):

$$c_1(x) = -1 + 2\frac{\int_{-t_1}^{x} p^{1/3}(x)dx}{\int_{-t_1}^{t_1} p^{1/3}(x)dx}, \quad -t_1 < x < t_1. \tag{11}$$

By analogy, the optimal compressor function $c_2(x)$ is given by:

$$c_2(x) = \begin{cases} -1 + 2\frac{\int_{-\infty}^{x} p^{1/3}(u)du}{\int_{-\infty}^{-t_1} p^{1/3}(u)du + \int_{t_1}^{+\infty} p^{1/3}(u)du}, & -\infty < x < -t_1 \\ -1 + 2\frac{\int_{-\infty}^{-t_1} p^{1/3}(u)du + \int_{t_1}^{x} p^{1/3}(u)du}{\int_{-\infty}^{-t_1} p^{1/3}(u)du + \int_{t_1}^{+\infty} p^{1/3}(u)du}, & t_1 < x < \infty \end{cases}. \tag{12}$$

Since the function $p(x)$ is symmetric, by direct evaluation we obtain, for every $t_1 > 0$, that:

$$\int_{-\infty}^{-t_1} p^{1/3}(x)dx = \int_{t_1}^{+\infty} p^{1/3}(x)dx = 3\sqrt[3]{\frac{\sigma}{4}} \exp\left(-\frac{\sqrt{2}t_1}{3}\right).$$

Hence for $t_1 > 0$, compressor functions c_1 and c_2 can be expressed as:

$$\begin{aligned} c_1(x) &= \begin{cases} -\frac{1-\exp\left(\frac{\sqrt{2}x}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)}, & -t_1 < x < 0 \\ \frac{1-\exp\left(-\frac{\sqrt{2}x}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)}, & 0 < x < t_1 \end{cases}, \\ c_2(x) &= \begin{cases} -1 + \exp\left(\frac{\sqrt{2}}{3\sigma}(x + t_1)\right), & -\infty < x < -t_1 \\ 1 - \exp\left(-\frac{\sqrt{2}}{3\sigma}(x - t_1)\right), & t_1 < x < +\infty \end{cases}. \end{aligned} \tag{13}$$

The total signal distortion D is given by $D = D_i + D_o$ where D_i and D_o are distortions for the inner and outer regions respectively. They can be approximated using Bennet integral as follows

$$D_i = \frac{2}{3N_1^2}\left(\int_0^{t_1} p^{1/3}(u)du\right)^3 = \frac{9\sigma^2\left(1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)\right)^3}{2N_1^2},$$
$$D_o = \frac{2}{3N_2^2}\left(\int_{t_1}^{+\infty} p^{1/3}(u)du\right)^3 = \frac{9\sigma^2\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)}{2N_2^2}. \qquad (14)$$

According to the last expression we see that total distortion D is the function of the parameters N_1, N_2 and t_1, i.e. we may write:

$$D = D(N_1, N_2, t_1) = \frac{9\sigma^2\left(1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)\right)^3}{2N_1^2} + \frac{9\sigma^2\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)}{2N_2^2}. \qquad (15)$$

Similarly the average number of bits per sample R is given by $R = p_1\log_2 N_1 + p_2\log_2 N_2$, where p_1 and p_2 are probabilities that one signal sample will belong to I and O respectively. Since

$$p_1 = \int_{-t_1}^{t_1} p(u)du = 1-\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right), \qquad (16)$$

and $p_2 = 1 - p_1$ we see that R can be also expressed as a function of N_1, N_2 and t_1 in the following way:

$$\begin{aligned} R &= R(N_1, N_2, t_1) \\ &= \left(1-\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)\right)(\log_2 N_1 + 1) \\ &+ \exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)(\log_2 N_2 + 1)\,. \end{aligned} \qquad (17)$$

The additional bit in expression (17) determines which quantizer is used in the coding process. This information is necessary for decoding. Note that threshold t_1 can be computed directly from the value R using:

$$t_1 = -\frac{\sigma}{\sqrt{2}}\ln\left(\frac{R-\log_2 N_1 - 1}{\log_2 N_2 - \log_2 N_1}\right). \qquad (18)$$

3.2 Three companding quantizers VLC model

The coder consists of three companding quantizers with different number of representation levels and different compressor functions. First quantizer Q_1 is applied on the inner segment, $I_1 = [-t_1, t_1]$, second quantizer Q_2 is applied on the second inner segment $I_2 = [-t_2, -t_1] \cup [t_1, t_2]$, while the third quantizer Q_3 is applied on the outer segment $O = (-\infty, t_2] \cup [t_2, +\infty)$. We say that values t_1, t_2 and t_3 are threshold values. We also denote by $N_i = 2^{k_i}$ the number of the quantization levels of quantizer Q_i, where k_i is number of bits per sample and $i = 1, 2, 3$. Let $c_1 : I_1 \to [-1, 1]$, $c_2 : I_2 \to [-1, 1]$ and $c_3 : O \to (-1, 1)$ be corresponding optimal compressor functions. These functions are given similarly as in the case of two quantizers (relations (11) and (12)):

$$c_1(x) = \begin{cases} -\frac{1-\exp\left(\frac{\sqrt{2}x}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)}, & -t_1 < x < 0 \\ \frac{1-\exp\left(-\frac{\sqrt{2}x}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)}, & 0 < x < t_1 \end{cases},$$

$$c_2(x) = \begin{cases} -\frac{1-\exp\left(\frac{\sqrt{2}(x+t_1)}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}(t_2-t_1)}{3\sigma}\right)} & -t_2 < x < -t_1, \\ \frac{1-\exp\left(-\frac{\sqrt{2}(x-t_1)}{3\sigma}\right)}{1-\exp\left(-\frac{\sqrt{2}(t_2-t_1)}{3\sigma}\right)} & t_1 < x < t_2, \end{cases},$$

$$c_3(x) = \begin{cases} -1 + \exp\left(\frac{\sqrt{2}}{3\sigma}(x+t_2)\right), & -\infty < x < -t_2 \\ 1 - \exp\left(-\frac{\sqrt{2}}{3\sigma}(x-t_2)\right), & t_2 < x < +\infty \end{cases}.$$

The total signal distortion D is given by $D = D_1 + D_2 + D_3$ where D_i is distortion for the inner or outer regions, respectively. The average number of bits per sample R is given by $R = p_1 \log_2 N_1 + p_2 \log_2 N_2 + p_3 \log_2 N_3$, where p_i are probabilities that one signal sample belongs to I_i or O, respectively. After some basic calculations, similarly to the previous case, we see that D and R can be expressed in the following way:

$$\begin{aligned} D(N_1,N_2,N_3,t_1,t_2) &= \frac{9\sigma^2\left(1-\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)\right)^3}{2N_1^2} \\ &+ \frac{9\sigma^2\left(\exp\left(-\frac{\sqrt{2}t_1}{3\sigma}\right)-\exp\left(-\frac{\sqrt{2}t_2}{3\sigma}\right)\right)^3}{2N_2^2} \\ &+ \frac{9\sigma^2\left(\exp\left(-\frac{\sqrt{2}t_2}{3\sigma}\right)\right)^3}{2N_3^2}, \end{aligned} \tag{19}$$

$$\begin{aligned} R(N_1,N_2,N_3,t_1,t_2) &= \left(1-\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)\right)(\log_2 N_1+1) \\ &+ \left(\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)-\exp\left(-\frac{\sqrt{2}t_2}{\sigma}\right)\right)(\log_2 N_2+2) \\ &+ \exp\left(-\frac{\sqrt{2}t_2}{\sigma}\right)(\log_2 N_3+2)\,. \end{aligned} \tag{20}$$

Extra one and two bits, respectively, are added in the rates of every used quantizers in the expression (20). They determine which quantizer is used in the coding process. This is the side information necessary for decoding. If the source sample belongs to region I_1, the first bit of the codeword is set to zero and the following bit rate $R_1 = \log_2 N_1 + 1$ is used. Otherwise, if source sample belongs to second or third segment, the first two bits of codeword are set to 10 or 11, respectively. Since other $\log_2 N_2$ and $\log_2 N_3$ are used for information, corresponding bit rates are $R_2 = \log_2 N_1 + 2$ and $R_3 = \log_2 N_1 + 2$. Such coding enables the simple decoder structure. Since the coder depends on two parameters t_1 and t_2, we perform an optimization for the fixed value of average bit rate. In other words, we solve the following optimization problem:

$$\begin{aligned} \min \quad & D \\ s.t. \quad & R = R_0 \\ & 0 \leq t_1 \leq t_2 \end{aligned}$$

The optimization procedure can be described as follows. First we express t_2 from (20), considering that $R = R_0$, and then replace it in (19). Hence we obtain the distortion D only as a function of t_1, i.e. $D(t_1)$. Then it can be minimized using one of the well-known unconstrained optimization methods (for example, Simplex method, variant I) [19].

The above given optimization is valid if thresholds follow relationship $0 \leq t_1 \leq t_2$ i.e.

$$\exp\left(-\sqrt{2}t_2/\sigma\right) \leq \exp\left(-\sqrt{2}t_1/\sigma\right) \leq 1. \tag{21}$$

Combining (20) and (21) we can derive the conditions for the cases represented in Table 1. It can be seen that there exist four cases with three conditions, respectively. Our method of optimization is correct for any of these cases. Following these conditions in the terms of t_1, k_1, k_2, k_3, and R_0, we have obtained a solution which provides a unique minimum value of D, which means that our method of optimization is correct. In Table 1. t_d and t_u are defined as:

$$t_d = -\frac{1}{\sqrt{2}} \ln \frac{k_1 + 1 - R_0}{k_1 - k_2 - 1}, \tag{22}$$

$$t_u = -\frac{1}{\sqrt{2}} \ln \frac{R_0 - k_1 - 1}{k_3 - k_1 + 1}. \tag{23}$$

and they denote lower and upper borders of range in which interval threshold t_1 can take its values to satisfy the constraints in optimization model.

	1st condition	2nd condition	3rd condition
1.	$t_d \leq t_1 \leq t_u$	$k_1 - 1 \leq k_2 \leq k_3$	$k_1 + 1 - R_0 \leq 0$
2.	$t_d \leq t_1 \leq t_u$	$k_1 - 1 \geq k_3 \geq k_2$	$k_1 + 1 - R_0 \geq 0$
3.	$t_d \leq t_1 \leq t_u$	$k_1 - 1 \geq k_2 \geq k_3$	$k_1 + 1 - R_0 \geq 0$
4.	$t_d \geq t_1 \geq t_u$	$k_1 - 1 \leq k_3 \leq k_2$	$k_1 + 1 - R_0 \leq 0$

Table 1: Four cases with three range conditions in each case as a function of threshold t_1, sample bit rates k_1, k_2, k_3, and R_0 for which our method of optimization is correct, respectively.

3.3 Block diagrams of coder and decoder

Block schemes of our VLC model are shown on Figure 1.

On the transmission side, input signal I first goes through quantizer selector where it is compared with the threshold values t_1 in the case of two ($i = 2$) and t_1, t_2 in the case of three quantizers ($i = 3$). Quantizer selector sends the signal to the corresponding companding quantizer (consisting of compressor and uniform quantizer). Output consists of the quantizer output O and the information about the selected quantizer (O_Q).

On the reception side, both information go to the decoder selector which sends the quantizer output O into the corresponding decoder (consisting of the uniform quantizer decoder and expandor), according to the information O_Q.

4 Numerical examples and optimization

The value often used for description of the quality of quantizer is *Signal to Quantizer Noise Ratio* ($SQNR$) defined by:

$$SQNR = 10 \log \left(\frac{\sigma^2}{D} \right) \tag{24}$$

In this section we use value of $SQNR$ to measure the performance of the quantizers, instead of the distortion D.

Numerical results corresponding to the first model (two quantizers) are shown in Figure 2. We plotted the value of $SQNR$ in relation to the average bit rate R for different values of N_1 and N_2. Every line is obtained as a parametric curve $(R(t_1), SQNR(t_1))$. Range of parameter t_1 is chosen in the way that $R(t_1)$ goes from its minimum to its maximum value. Optimization is performed by varying different values of k_1 and k_2 and computing $SQNR$ where $R = R_0$ is fixed.

Figure 3 shows the numerical results corresponding to the second model (three quantizers). Value of $SQNR$ is plotted in relation to the average bit rate R for different numbers of representation levels N_1, N_2 and N_3. Optimal dependency is practically linear with slope value equal to 6, i.e. $SQNR$ increase 6dB per bit.

Comparison of quantization models with one, two, and three regions is given in Figure 4. The increase in $SQNR$ between methods with one and two regions is 2.8dB, and between methods with one and three regions is 4.2dB approximately, in favor of our multi-resolution scalar quantizer. We also made a comparison between our model and coder consisting of the optimal uniform quantizer and Huffmann lossless coder. We assume that the uniform quantizer $Q_u(x)$ is applied in the support region $[-t_{max}, t_{max}]$ where the bound t_{max} is optimized. From Figure 4 we can observe that our method outperforms the well-known mentioned methods. Comparation is also made with the combination of optimal companding quantizer and simple lossless coder [16]. Our model has more complex quntizer, but simpler coder for two quantizer model reaches gain of 1.3dB. Considering model with three regions we reach gain of 2.7dB, with slightly more complex realization structure.

5 Application in speech coding

We have tested our coding scheme on the speech coding. The sample signal is taken from the base which is derived from the TIMIT corpus [22]. The TIMIT corpus of speech has been designed to provide speech data for the acquisition of acoustic-phonetic knowledge and for the development and evaluation of automatic speech recognition systems.

For the purpose of testing, we consider the adaptive variant of our three quantizer VLC model. The block scheme is given in the Figure 5.

The original VLC model assumes that the input signal is Laplacian source with variance (power of the signal) equal to $\sigma = 1$. In general, speech signal can be approximated by Laplacian source with variable variance. Hence, we divide the input signal into the frames and for each frame we estimate the signal variance and normalize all samples before coding.

Consider the n samples of the input signal $x_1, x_2, \ldots, x_n$ and assume that signal samples are divided in F frames and each frame consist of M samples. Furthermore denote by $x_{i,j}$ the j-th sample of the i-th frame ($i = 0, \ldots, F-1$ and $j = 0, \ldots, M-1$), i.e. $x_{i,j} = x_{iM+j}$. In the i-th frame signal variance is estimated using $\sigma_i^2 = \frac{1}{M}\sum_{j=0}^{M-1} x_{i,j}^2$. The source samples are then normalized and sent to the quantizer. When received, the signal has to be denormalized. For that purpose, we also need to transmit the signal variance σ_i. It is quantized using log-uniform quantizer with N_g levels and sent to the beginning of the each frame. Other signal samples are normalized to $\bar{x}_{i,j} = x_{i,j}/\tilde{\sigma}_i$, where $\tilde{\sigma}_i$ is quantized signal variance, and then sent to the quantizer.

The representation levels and decision thresholds of log-uniform quantizer $Q_{lu}(\sigma)$ are defined as:

$$\log(y_{lu,i}) = \log(\sigma_{min}) + \frac{2i-1}{2N_g} \log\frac{\sigma_{max}}{\sigma_{min}}, \quad i = 1, 2, \ldots, N_g,$$
$$\log(t_{lu,i}) = \log(\sigma_{min}) + \frac{i}{N_g} \log\frac{\sigma_{max}}{\sigma_{min}}, \quad i = 0, 1, 2, \ldots, N_g,$$

where σ_{min} and σ_{max} are respectively maximum and minimum possible value of the signal variance. In other words, log-uniform quantizer is the uniform quantizer in decibels scale. We used the dynamic range of the variance ($20\log(\sigma_{max}/\sigma_{min})$[dB]) of 40dB and $N_g = 32$ levels of log-uniform quantizer. In the Figure 6 we show the $SQNR$ value as the function of σ for adaptive variant of three quantizer VLC model and different values of N_g.

Value $N_g = 1$ corresponds to the non-adaptive case, i.e. when input signal goes directly to quantizer. Note that $SQNR$ value is not attending its maximum at point $\sigma = 0$dB (it is attending at $\sigma^* = 2.45$dB). However, varying σ violates the condition $R = R_0$, i.e. bit rate R is also changing. Therefore, we have to adapt variance to initial value $\sigma = 0$dB ($\sigma = 1$). As it can be seen in the Figure 6, $SQNR$ value is almost constant for $N_g = 32$. Hence $N_g = 32$ is good choice for the number of levels of variance quantizer.

For the purpose of the experiment, we choose the frame size $M = 200$ and total $F = 800$ frames. We determine the experimental value $SQNR_i^{ex}$ for each frame $i = 0, 1, \ldots, F-1$. In Figure 7 we show the input signal (upper graph) and $SQNR_i^{ex}$

R	k_1	k_2	k_3	t_1	t_2	$SQNR$	$SQNR^{ex}$
11	9	10	12	0.636	1.677	63.7116	63.7187
10.5	8	9	11	0.419	1.148	60.754	61.1961
10	8	8	10	0.467	1.004	57.0683	57.3965
9.5	7	8	10	0.419	1.148	54.7334	55.1332
9	7	7	10	0.574	1.192	51.9223	52.0192

Table 2: Comparison between theoretical and experimental value of $SQNR$ of optimal three quantizer VLC model, for different values of bitrate R.

values (lower graph). Experiment is done for $R = 9$ and the corresponding optimal three quantizer VLC model (parameters are $k_1 = k_2 = 7$, $k_3 = 10$, $t_1 = 0.574$ and $t_2 = 1.192$).

Note that the x scale on lower graph still represents the index of the sample (not the frame). The average $SQNR$ value of all frames is equal to $SQNR^{ex} = \frac{1}{F}\sum_{i=0}^{F-1} SQNR_i^{ex}$. In our case it follows that $SQNR^{ex} = 52.0192$ and theoretical value is $SQNR = 51.9223$ (from Figure 3 and Figure 4). Hence, we obtain good agreement between theory and experiment in this case. We compared theoretical and experimental values of $SQNR$ for the several different values of the bitrate R. As it can be seen from Table 2, there is a good agreement between theory and experiment in all cases.

6 Conclusion

This paper provides the simple structure coder for memoryless Laplacian source. We have used companding model based on the two companding quantizers in the first case, and then the three companding quantizers in the second case, with different number of representation levels and different compressor functions. There are analytical estimates of the distortion, average bit rate and signal to quantization noise ratio derived. We have also performed the R-D optimization for both two and three quantizers case. Generally our method gives the very simple realization structure and performances close to optimal ones and hence it is very useful in practical applications, such as speech signals, images, etc. That is the main advantage of our model. We have also made a comparison between our model and coder consisting of the uniform quantizer (with optimal support range) and Huffmann lossless coder Second comparison has bean made between our model and combination of optimal bounded companding quantizer and simple lossless coder. It is shown that our coder possesses better results, with much simple and efficient realization structure. Theoretical results are verified by the experiment on the sample speech signal. The above discussion points to the fact that our method outperforms well-known mentioned methods.

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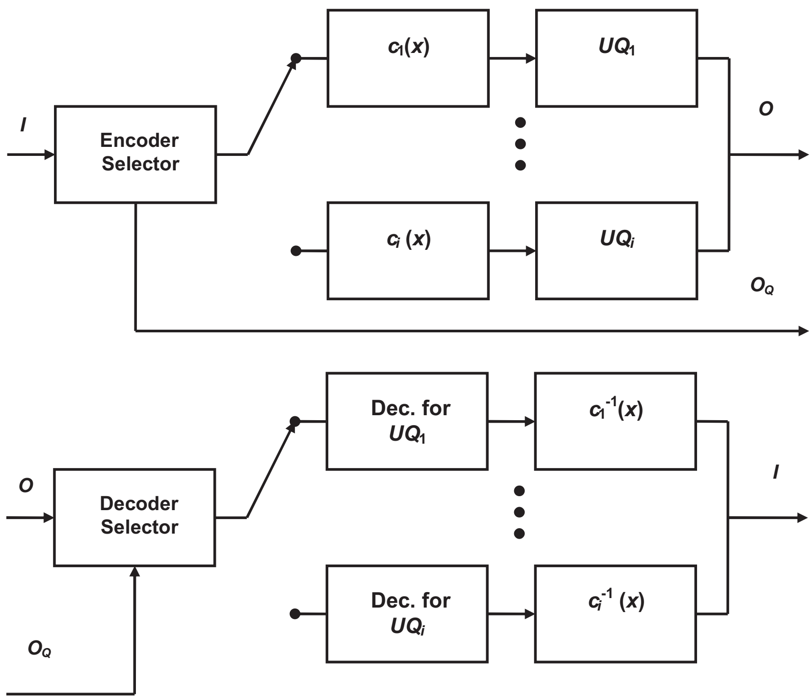

Figure 1: Block schemes of coder (upper scheme) and decoder (lower scheme), corresponding to two ($i = 2$) and three ($i = 3$) quantizer VLC model

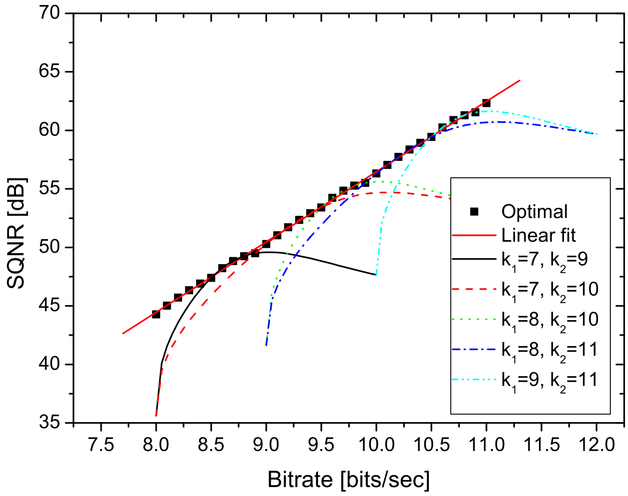

Figure 2: $SQNR$ in relation to the average bit rate R for different numbers of quantization points in each of two quantizers.

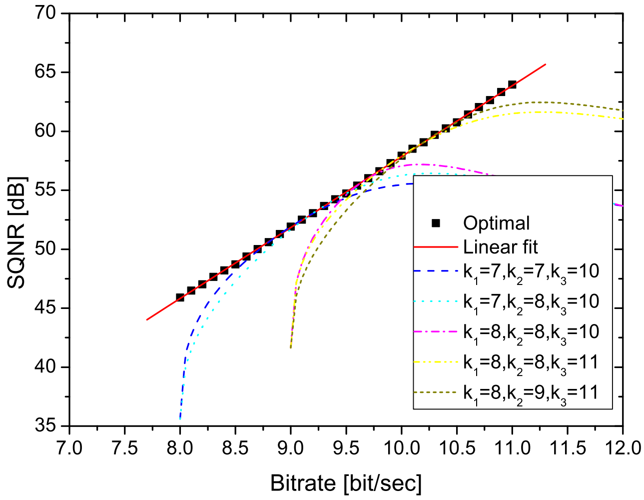

Figure 3: $SQNR$ in relation to the average bit rate R for different numbers of quantization points in each of three quantizers.

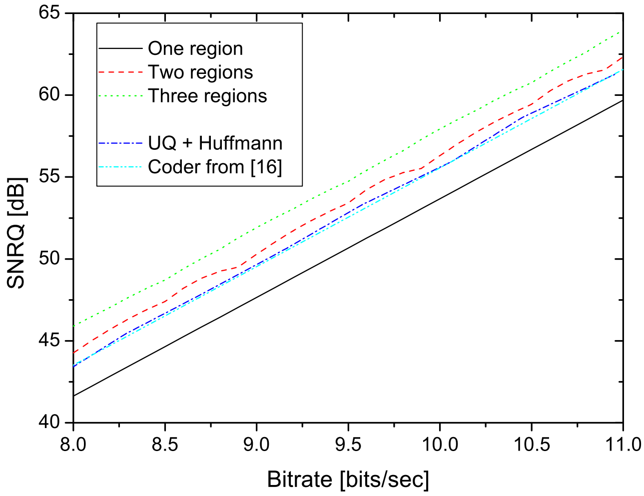

Figure 4: $SQNR$ in relation to the average bit rate R for quantization models with one, two, three regions, combination of the optimal uniform quantizer and Huffmann lossless coder and combination of optimal bounded companding quantizer and simple lossless coder [16]

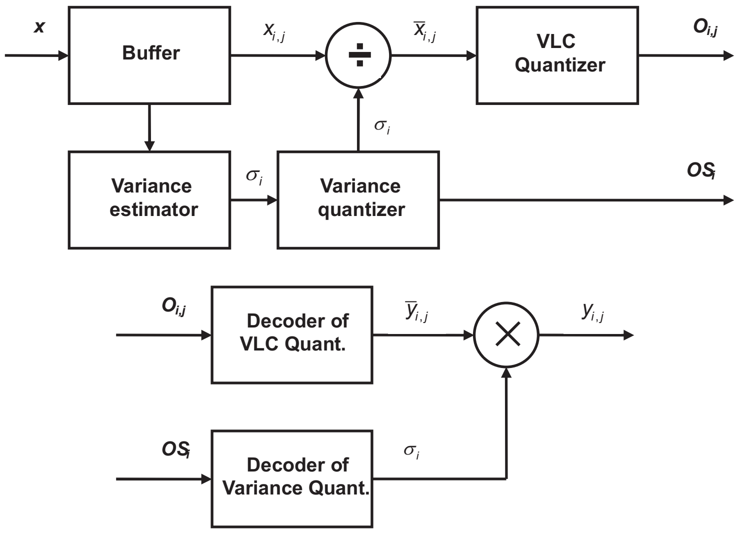

Figure 5: Block scheme of adaptive VLC quantizer: transmitter (upper scheme) and receiver (lower scheme).

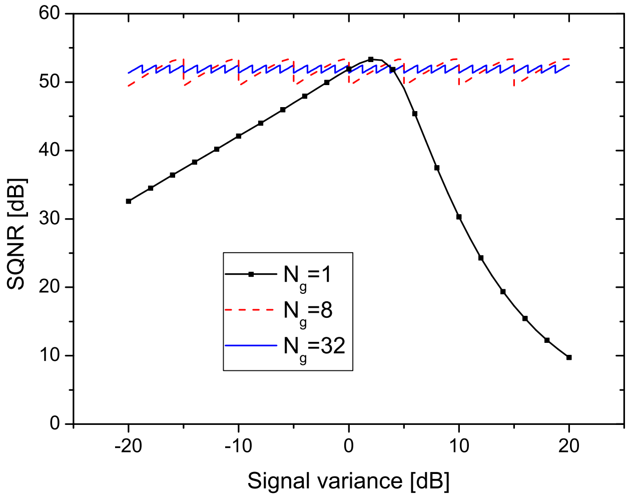

Figure 6: Theoretical dependence of the $SQNR$ value as a function of signal variance σ (in decibels) for non-adaptive ($N_g = 1$) and adaptive ($N_g = 8, 32$) three quantizer VLC model. Parameters of VLC model are equal to $k_1 = k_2 = 7$, $k_3 = 10$, $t_1 = 0.574$ and $t_2 = 1.192$.

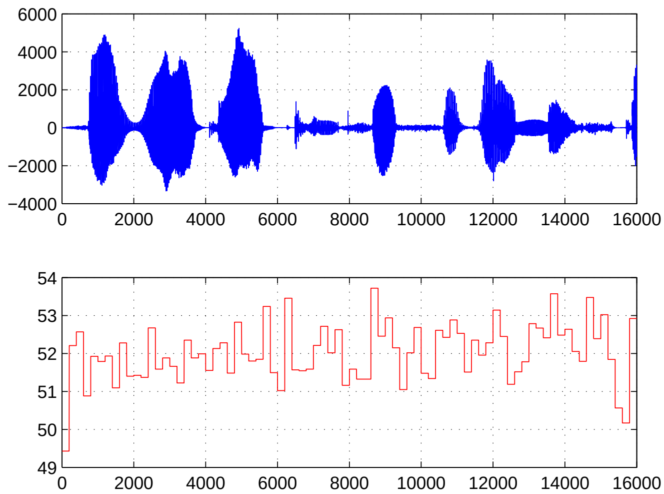

Figure 7: An input signal (upper graph) and experimental value of $SQNR$ (lower graph) for the optimal three quantizer VLC model with $R = 9$.