Preface

At the beginning of the last century, many scientific disciplines recognized the need for description of real-life phenomena which are time-dependent and affected by stochastic excitation, and which are, therefore, modelled mathematically by using stochastic processes. A whole lot of mathematicians headed by Kolmogorov, Markov, Hintchin, Cramer, and Gikhman, as well as physicists and engineers including Einstein, Peren, and Weiner, made an enormous contribution to the establishment and development of the theory of stochastic processes. The concept of a stochastic process has thus proved to be a central one not only in the stochastic theory, but also in various applied disciplines such as mechanics, engineering, economics, production management, etc., that is, in all areas where randomness occurs as an overall phenomenon. It is due to this tight intertwinement with the practical affairs of life that the theory of stochastic processes is considered one of the most rapidly developing mathematical disciplines.

A distinctive position in the theory of stochastic processes is occupied by stochastic differential equations in which randomness phenomena are represented by the Brownian motion. The fundamentals of the theory of stochastic differential equations were postulated in the 1940s, independently by the Russian mathematician I.I. Gikhman and the Japanese mathematician N. Ito. It is Ito's approach, built up on the definition of stochastic integrals relative to the Brownian motion, that is generally assumed today. Since then, the development of the theory has been rapid and continuous, and frequently facilitated by problems in those areas in which dynamic phenomena are stochastically modelled and described.

A large number of scientific papers, books and monographs have so far been concerned with the various types of stochastic differential equations, with the primary aim of studying their solvability and the properties of solutions. However, the number of effectively solvable stochastic differential equations is still insignificant due to the fact that, in a majority of cases, the analytic form of the solution cannot be found. In such cases, analytic or numerical methods are commonly employed, that is, searching for approximate solutions which are, in some sense, close to the solution to the initial equation. It is to some of the analytic methods for determination of approximate solutions that this monograph is devoted. It has resulted from our doctoral dissertations and the papers published within the last few years.

The book is intended for master and doctoral students with a prior acquaintance with stochastic processes and stochastic differential equations. Researchers in applied disciplines, such as mechanics, engineering, economics and biomedicine, may also find it helpful.

The book consists of nine chapters. The first two chapters are of the introductory character and contain the basic notions and definitions used in the theory of stochastic processes and stochastic differential equations.

Chapters 3-5 refer to perturbed stochastic differential and integrodifferential equations. In Chapter 3 we present the basic idea behind the study of the effect which perturbations dependent on small parameters have upon the solutions of some classes of additive perturbed stochastic differential and integrodifferential equations. Solutions to perturbed equations and solutions to corresponding unperturbed equations of the same or simpler type in the (2m)-th mean are compared on finite intervals and on intervals whose length tends to infinity as small parameters tend to zero. In this respect, it seems logical to understand solutions to unperturbed equations as analytic approximations of solutions to more complex, perturbed equations. In Chapter 4, perturbed stochastic differential and integrodifferential equations with linear perturbations are investigated. Chapter 5 is devoted to functionally perturbed stochastic differential equations, that is, the general form of perturbations that are in an arbitrary functional relation with the coefficients of the initial equation.

Chapters 6-7 are concerned with hereditary stochastic integrodifferential equations, that is, stochastic equations defined on a functional space with a "memory". In Chapter 6, the problems of the existence and uniqueness of solutions are studied, based on the notion of a random integral contractor. In Chapter 7, additive perturbed hereditary stochastic differential and integrodifferential equations with Lipschitz condition and random integral contractors are studied. The closeness in the 2m-th mean of perturbed and corresponding unperturbed hereditary equations is studied, on finite intervals and on intervals the length of which tends to infinity when small perturbations tend to zero. This chapter also contains some limit theorems resulting from the discretization of the small parameter.

In Chapter 8 a general analytic iterative method – the Z-algorithm – is described that is used for obtaining approximate solutions to stochastic differential and integrodifferential equations. The sequences of iterations converge with probability one to the solution of the initial equation. Some specific stochastic approximations are presented, based on the linearization of the coefficients of the initial equation. It is also shown that iterative procedures for the existence of solutions in Chapter 6 are just special cases of the Z-algorithm.

Chapter 9 is concerned with a special analytic approximation for stochastic differential and integrodifferential equations. Approximations are obtained by successive connection of solutions to approximate equations defined on a partition of the time-interval. The coefficients of the approximate equations represent Taylor series of the arbitrary order for the coefficients of the initial equation. The closeness of the initial and approximate equations is measured in the p-th mean and in probability one. This form of approximation can be significant for the application of numerical methods, given that polynomials are highly convenient mathematical objects for numerical approximations.

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