Contents

1 Essentials of the theory of stochastic processes **1**
1.1 Brief survey of stochastic processes 1
1.1.1 Definitions and preliminary considerations 2
1.1.2 Filtration 5
1.1.3 Basic classes of stochastic processes 6
1.1.4 Martingales 8
1.2 Brownian motion 10
1.3 Gronwall–Bellman type inequalities 17

2 Essentials of stochastic integrals and stochastic differential equations **19**
2.1 Introduction 19
2.2 The Itô calculus 21
2.2.1 The Itô stochastic integral 21
2.2.2 The indefinite Itô integral 29
2.2.3 The Itô formula 32
2.2.4 Moment inequalities with Itô integrals 36
2.3 The Itô stochastic differential equations 38
2.3.1 Definitions and basic notation 38
2.3.2 Existence and uniqueness of solutions 41
2.3.3 The solution as a Markov process 51
2.3.4 Linear stochastic differential equations 52
2.3.5 Stochastic differential equations dependent on parameters 56
2.4 Stochastic integrodifferential equations 59
2.4.1 Stochastic integrodifferential equations linear with respect to integrals 59
2.4.2 General stochastic integrodifferential equations 61

3 Stochastic differential equations with additive perturbations **65**
3.1 Introduction and preliminary results 65
3.2 Additively perturbed stochastic integrodifferential equations . . . 70
3.3 Additively perturbed general stochastic integrodifferential equations 83
3.4 An application in mechanics 93

4 Stochastic differential and integrodifferential equations with linear perturbations **99**
4.1 Stochastic differential equations with linear perturbations 99
4.2 General stochastic integrodifferential equations with linear perturbations 104

5 Stochastic differential equations with functional perturbations **121**
5.1 Functionally perturbed stochastic differential equations 121
5.1.1 Numerical simulation 133

6 Existence and uniqueness problems for stochastic hereditary integrodifferential equations **139**
6.1 Introduction and preliminary results 139
6.2 Stochastic hereditary integrodifferential equations via integral contractors 144
6.3 Comparison theorems 158

7 Perturbed stochastic hereditary differential and integrodifferential equations **169**
7.1 Perturbed stochastic hereditary integrodifferential equations with Lipschitz conditions 169
7.2 Perturbed stochastic hereditary differential equations with integral contractors 180

8 Some analytic methods for solving stochastic differential equations **191**
8.1 Introduction and preliminary results 191
8.2 The Z-algorithm for solving stochastic differential equations . . . 193
8.3 The Z-algorithm for solving stochastic hereditary integrodifferential equations 202
8.4 The Z-algorithm and the contractor theory 211

9 Analytic approximations of solutions via Taylor series **215**
9.1 Introduction and preliminary results 215
9.2 Analytic approximations of solutions to stochastic differential equations 217
9.3 Analytic approximations of solutions to stochastic integrodifferential equations 226

Bibliography **239**