Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ with constant sectional curvature

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Nearly Kähler manifolds have been studies intensively in the 1970s by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is nonintegrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on $\mathbb{R}^7$. Recently it has been shown by Butruille that the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6- sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Totally real submanifolds are those for which the almost complex structure maps tangent vectors to normal vectors. A special case occurs when the dimension of the submanifold is half of the dimension of the ambiant space. In that case such submanifolds are called Lagrangian and J interchanges the tangent and the normal space. In this talk we study Lagrangian submanifolds of $S^3 \times S^3$. The first examples of such Lagrangian submanifolds were due to Schäfer and Smoczyk. Other examples have been recently discovered by Moroianu and Semmelmann. In this talk we obtain several classification results regarding such submanifolds. These results include the classification of the totally geodesic ones as well as the classification of the ones with constant sectional curvature.

Joint work with B. Dioos (Leuven) and X. Wang (Nankai).