The geometry of iterations defined on subsets of measure metric space and its applications to the tumor spread problem

Elena Vilkul

Lomonosov Moscow State University, Moscow, RUSSIA
[elena.tuzhilina@mail.ru]

Researches devoted to cancer have occupied a key position in biology and medicine during last few decades. One of the most interesting problems related to this topic is connected to the cancer spread modeling. Usually model of tumor evolution is described as an iterative process. It means that there is an initial state of the tissue discribed as a set of ill and healthy cells, and each subsequent state is determined by the previous one. Moreover, it is assumed that the tumor spreads by a certain law and that the state of each cell is defined by the surrounding cells.

There are a large number of articles devoted to the discrete model of the cancer spread. Such models consider the tissue as $\mathbb{Z}^2$ and identify every element of $\mathbb{Z}^2$ with a cell. In the present talk we discuss more general model including the continuous case that describes the tissue as a metric space (X, d) with the cells corresponding to the points in this metric space. Suppose that there is a measure μ defined on the considered metric space, in fact, X is a measure metric space (mm-space). Consider a set $B_0 \subset X$ that relates to an initial configuration of the ill cells. The iterative process is determined by the following rule. Choose a real positive number r and some $k \in [0, 1)$. A point $x \in X$ becomes infected on the next step if the closed ball $B(x, r)$ with the center x and radius r contains at least k percent of the ill cells.

For the formal description of the iterative tumor evolution we introduce a function $P_r(A, x) = \frac{\mu(A \cap B(x,r))}{\mu(B(x,r))}$. Function $F(A) = \{x \in X : P_r(A, x) \geq k\}$ maps the set of all subsets of X to itself and characterizes the process of cancer spread. Tumor evolution with the initial state B_0 is described by the sets sequence $B_{i+1} = F(B_i)$. The first problem of the research is connected to properties desctiption of the mapping F and its iterations. For example, the stationary points of this mapping can be interpreted as the insistent tumors. We can define the notion of a curable tumor as a set B_0 such that $\mu(B_i) \to 0$ as $i \to \infty$. The second problem under consideration is classification of Borel sets in terms of curability. This talk will present several

results connected with the model discussed above and describe some open-problems in this field.
