Pontryagin algebras of some moment-angle complexes

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We consider the problem of describing the Pontryagin algebra (loop homology) of moment-angle complexes and manifolds. The moment-angle complex $\mathcal{Z}_{\mathcal{K}}$ is a cell complex built of products of polydiscs and tori parametrised by simplices in a finite simplicial complex $\mathcal{K}$. It has a natural torus action and plays an important role in toric topology. In the case when $\mathcal{K}$ is a triangulation of a sphere, $\mathcal{Z}_{\mathcal{K}}$ is a topological manifold, which has interesting geometric structures.

Generators of the Pontryagin algebra $H_*(\Omega\mathcal{Z}_{\mathcal{K}})$ when $\mathcal{K}$ is a flag complex were described in the work of Grbic, Panov, Theriault and Wu. Describing relations is often a difficult problem, even when $\mathcal{K}$ has a few vertices. Here we describe these relations in the case when $\mathcal{K}$ is the boundary of pentagon or hexagon. In this case, it is known that $\mathcal{Z}_{\mathcal{K}}$ is a connected sum of products of spheres with two spheres in each product. Therefore $H_*(\Omega\mathcal{Z}_{\mathcal{K}})$ is a one-relator algebra and we describe this one relation explicitly, therefore giving a new homotopy-theoretical proof of McGavranś result. An interesting feature of our relation is that it includes iterated Whitehead products, which vanish under the Hurewicz homomorphism. Therefore, the form of this relation cannot be deduced solely from the result of McGavran.