

Bi-Hermitian metrics on Kato surfaces

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The problem of classification of bi-Hermitian structures on four-manifolds was introduced by S. Salamon in the 90's: given a fixed compact Hermitian surface (S, g, J) - where g denotes a Riemannian metric, is it possible to find a different complex structure I which is still Hermitian with respect to the same metric g ? The first instance of this phenomenon are hypercomplex structures, for example hyperKähler surfaces in which case however one actually gets an infinite number of such I 's. The problem of finding and classifying bi-Hermitian surfaces is clearly a conformally invariant one and has proved to be connected with at least two interesting fields of active research in complex geometry. The first one is more classical - initiated by I. Vaisman and F. Tricerri - and is the problem of finding locally conformally Kähler structures, usually abbreviated with l.c.K. The second one is more recent - due to N. Hitchin and M. Gualtieri - who introduced generalized Kähler geometry in the early 2000's and showed that such a structure always gives rise to a bi-Hermitian structure.

Compact Kähler surfaces are naturally divided in two classes: surfaces of Kähler type and non-Kähler type. As the Kähler case has recently been classified by the work of Hitchin, Gualtieri and Goto, our main contributions are concerned with the non-Kähler case.

Our main contributions are the following:

1. A complete classification of surfaces admitting a bi-Hermitian structure such that the anti-canonical divisor is disconnected. This was achieved via a twistor construction also produced as a by-product new antiself-dual metrics as well as new l.c.K. metrics, stimulating novel interest and ideas in the subject eventually leading to a power result of M. Brunella who solved the l.c.K. existence question for all known compact complex surfaces.

2. A complete classification of bi-Hermitian surfaces with connected anti-canonical divisor. This new result is achieved thanks to the interplay - discovered by V. Apostolov-G. Dloussky - between l.c.K. metrics and bi-Hermitian structures.

The existence problem is now completely solved for all surfaces except for a class of Kato surfaces which are called *intermediate* for which our existence result is complete only up to a logarithmic deformation. We complete the classification of compact complex surfaces with two different complex structures orthogonal to a fixed Riemannian metric by considering the case of Kato surfaces admitting a connected numerical anti-canonical divisor.

Joint work with Prof. A. Fujiki.
