Lagrangian submanifolds in the nearly Kähler $S^3 \times S^3$ from minimal surfaces in S^3

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Nearly Kähler (abbrev. NK) manifolds are almost Hermitian manifolds with almost complex structure J satisfying that $\tilde{\nabla}J$ is skew-symmetric. Butruille [1] proved that the only homogeneous 6-dimensional NK manifolds are the NK $\mathbb{S}^6$, $\mathbb{S}^3 \times \mathbb{S}^3$, the complex projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. A natural and interesting question for the above four NK manifolds is to investigate their almost complex submanifolds and their Lagrangian submanifolds. As NK manifolds are an important class of Hermitian manifolds, we can consider Lagrangian submanifolds more generally in almost Hermitian manifolds. We say that such a submanifold is Lagrangian if the almost complex structure J interchanges the tangent and the normal spaces and if the dimension of the submanifold is half the dimension of the ambient manifold.

In the present talk, we refer to the study of minimal Lagrangian submanifolds M in the NK $\mathbb{S}^3 \times \mathbb{S}^3$ described by $g \mapsto f(g) = (p(g), q(g))$. It is known that in order to describe such submanifolds the so called angle functions (see [2]) play an important role.

Here we describe the Lagrangian submanifolds of the NK $\mathbb{S}^3 \times \mathbb{S}^3$ whose angle functions are $\theta_1 = \frac{\pi}{3}$, $\theta_2 = \alpha + \frac{\pi}{3}$ and $\theta_3 = -\alpha + \frac{\pi}{3}$. Geometrically this corresponds to the Lagrangian immersions for which the map p is not an immersion.
We show that M can be identified with an open part of a frame bundle over a minimal surface N. Moreover the immersion is determined by an additional differential equation. Moreover, we study as well the reverse problem, in the cases when the minimal surface is totally geodesic or not.

1. Butruille JB. Homogeneous nealy Kähler manifolds. Handbook of pseudo-Riemannian geometry and supersymmetry. IRMA Lectures in Mathematics and Theoretical Physics. European Mathematical Society. 2010; 16: 399-423.
2. Dioos B, Vrancken L, Wang X. Lagrangian submanifolds in the Nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$