Homotopy invariants of manifolds: Poincare duality and the signature formulae

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The Hirzebruch formula connects the signature of the manifold with some characteristic classes of the manifold. Namely the Hirzebruch formula has the following form:

$$\mathbf{sign}\, X = 2^{2k} \langle L(X), [X] \rangle ,$$

where the class $L(X)$ is so called multiplicative Hirzebruch genus defined by

$$L(X) = \prod_j \frac{t_j/2}{\tanh(t_j/2)},$$

where t_j are formal generators such that $\sigma_k(t_1^2, \dots, t_n^2) = p_k(X)$ where $p_k(X)$ are the Pontryagin classes.

The crucial property of the Hirzebruch formula is that its left-hand member is expressed exclusively in homotopy terms whereas its right-hand member is an invariant of the smooth structure of the manifold.

This means that if two smooth manifolds are homotopy equivalent and may have different Pontryagin classes nevertheless the Hirzebruch number has the same value.

This circumstance generated many natural problems, some of them resulted deep theorems whereas others are still open.

Inter alia I will speak on Novikov's conjecture (1970) about homotopy invariance of higher signatures and on a short natural proof of the Novikov's theorem (1965) about topological invariance of rational Pontryagin classes which was presented by M.Gromov (1995) on the base of homotopy invariance of some higher signatures. Among last results we will speak how to construct non commutative (symmetric) signature on non compact manifolds with proper action of a discrete group.

This is a joint work with Th. Yu. Popelensky.