Reillyś type inequality for the Laplacian associated to a density related with shrinkers for MCF

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Let $(\overline{M}, \langle,\rangle, e^{\psi})$ be a Riemannian manifold with a density, and let M be a closed n-dimensional submanifold of $\overline{M}$ with the induced metric and density. We give an upper bound on the first eigenvalue λ_1 of the closed eigenvalue problem for Δ_ψ (the Laplacian on M associated to the density) in terms of the average of the norm of the vector $\vec{H}_\psi + \overline{\nabla}\psi$ with respect to the volume form induced by the density, where $\vec{H}_\psi$ is the mean curvature of M associated to the density e^{ψ}.
When $\overline{M} = \mathbb{R}^{n+k}$ or $\overline{M} = S^{n+k-1}$, the equality between λ_1 and its bound implies that e^{ψ} is a Gaussian density ($\psi(x) = \frac{C}{2}|x|^2$, $C < 0$), and M is a shrinker for the mean curvature flow (MCF) on $\mathbb{R}^{n+k}$.
We prove also that $\lambda_1 = -C$ on the standard shrinker torus of revolution.
Based on this and on the Yauś conjecture on the first eigenvalue of minimal submanifolds of S^n, we conjecture that the equality $\lambda_1 = -C$ is true for all the shrinkers of MCF in R^{n+k}.
This is a joint work with M. Carmen Domingo-Juan.