Topology of isoenergy surfaces for Kovalevskaya integrable case on Lie algebra so(4)

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Let us consider the six-dimensional space $\mathbb{R}^6(\mathbf{J}, \mathbf{x})$ and the following one- parameter family of Poisson brackets depending on the real parameter κ:

$$\{J_i, J_j\} = \varepsilon_{ijk} J_k, \quad \{J_i, x_j\} = \varepsilon_{ijk} x_k, \quad \{x_i, x_j\} = \kappa \varepsilon_{ijk} J_k.$$

These brackets have two Casimir functions:

$$f_1 = x_1^2 + x_2^2 + x_3^2 + \kappa(J_1^2 + J_2^2 + J_3^2), \qquad f_2 = x_1 J_1 + x_2 J_2 + x_3 J_3.$$

I.V. Komarov in his paper [4] showed that the Kovalevskaya integrable case in rigid body dynamics can be included in a one-parameter family of integrable Hamiltonian systems on this pencil of Lie algebras $\mathrm{so}(4) - \mathrm{e}(3) - \mathrm{so}(3,1)$. The Kovalevskaya top was realized as a system on Lie algebra $\mathrm{e}(3)$. The Hamiltonian H and first integral have the following form:

$$H = J_1^2 + J_2^2 + 2J_3^2 + 2c_1 x_1,$$

$$K = (J_1^2 - J_2^2 - 2c_1 x_1 + \kappa c_1^2)^2 + (2J_1 J_2 - 2c_1 x_2)^2,$$

where c_1 is an arbitrary constant.

In the case of $\varkappa > 0$, $a > 0$ the common level surfaces $M^4_{a,b} = f_1 = a, f_2 = b$ of Casimir functions are compact orbits of coadjoint representation and symplectic leaves of the Lie- Poisson bracket. Every regular $M^4_{a,b}$ has a structure of Liouville foliation. Every two- dimensional torus is a closure of trajectories of this system. The purpose of topological analysis is the calculation of Fomenko-Zieschang invariant for 3-dimensional isoenergy surfaces ([1]). We continue I.K. Kozlov's research of this system. In [3] the bifurcation diagrams were constructed and nondegenerate critical points of the rank 0 were described. Some new results will be presented including the list of all types of isoenergy surfaces of the system.

[1] Fomenko AT, Bolsinov AV. Integrable Hamiltonian Systems: Geometry, Topology, Classification. CRC Press. 2004.

[2] Bolsinov AV, Richter PH, Fomenko AT. The method of loop molecules and the topology of the Kovalevskaya top. Matematicheskii Sbornic 2. 2000; 191: 151-188.

[3] Kozlov IK. The topology of the Liouville foliation for the Kovalevskaya integrable case on the Lie algebra $so(4)$. Sbornik: Mathematics. 2014; 205(4): 532-572.

[4] Komarov IV. Kowalewski basis for the hydrogen atom. Theoretical and Mathematical Physics. 1981; 47(1): 320-324.