Topology of isoenergy surfaces for Kovalevskaya integrable case on Lie algebra \(\text{so}(4)\)

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Let us consider the six-dimensional space \(\mathbb{R}^6(J,x)\) and the following one-parameter family of Poisson brackets depending on the real parameter \(\kappa\):

\[
\{J_i, J_j\} = \varepsilon_{ijk} J_k, \quad \{J_i, x_j\} = \varepsilon_{ijk} x_k, \quad \{x_i, x_j\} = \kappa \varepsilon_{ijk} J_k.
\]

These brackets have two Casimir functions:

\[
f_1 = x_1^2 + x_2^2 + x_3^2 + \kappa (J_1^2 + J_2^2 + J_3^2), \quad f_2 = x_1 J_1 + x_2 J_2 + x_3 J_3.
\]

I.V. Komarov in his paper [4] showed that the Kovalevskaya integrable case in rigid body dynamics can be included in a one-parameter family of integrable Hamiltonian systems on this pencil of Lie algebras \(\text{so}(4) - e(3) - \text{so}(3,1)\). The Kovalevskaya top was realized as a system on Lie algebra \(e(3)\). The Hamiltonian \(H\) and first integral have the following form:

\[
H = J_1^2 + J_2^2 + 2J_3^2 + 2c_1 x_1, \\
K = (J_1^2 - J_2^2 - 2c_1 x_1 + \kappa c_1^2)^2 + (2J_1 J_2 - 2c_1 x_2)^2,
\]

where \(c_1\) is an arbitrary constant.

In the case of \(\kappa > 0, \ a > 0\) the common level surfaces \(M_{a,b} = f_1 = a, f_2 = b\) of Casimir functions are compact orbits of coadjoint representation and symplectic leaves of the Lie-Poisson bracket. Every regular \(M_{a,b}\) has a structure of Liouville foliation. Every two-dimensional torus is a closure of trajectories of this system. The purpose of topological analysis is the calculation of Fomenko-Zieschang invariant for 3-dimensional isoenergy surfaces ([1]). We continue I.K. Kozlov’s research of this system. In [3] the bifurcation diagrams were constructed and nondegenerate critical points of the rank 0 were described. Some new results will be presented including the list of all types of isoenergy surfaces of the system.

