On the existence of pre-semigeodesic coordinates

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In the present lecture we consider the problem of the existence of presemigeodesic coordinates on manifolds with affine connection. We proved that pre-semigeodesic coordinates exist in the case when the components of the affine connection are differentiable functions.

Let $A_n = (M, \nabla)$ be an n-dimensional manifold M with the affine connection ∇, dimension $n \geq 2$, and let $U \subset M$ be a coordinate neighbourhood at the point $x_0 \in U$. A couple (U, x) is a coordinate map on A_n.

Semigeodesic coordinate systems on surfaces and (pseudo-) Riemannian manifolds are generalized in the following way [2]:

Definition 1. Coordinates (U, x) in A_n are called *pre-semigeodesic coordinates* if one system of coordinate lines is geodesic and the natural parameter is just the first coordinate.

In [1, 2] the following theorems were proved.

Theorem 1. *The conditions* $\Gamma^h_{11}(x) = 0$, $h = 1, \dots, n$, *are satisfied in* (U, x) *if and only if* (U, x) *is pre-semigeodesic.*

We thought that the existence of this chart is trivial. This problem is obviously more difficult than we supposed. This was observed in [3] where precisely the existence of pre-semigeodesic charts was proved in the case when the components of the affine connection are real analytic functions.

We proved that the pre-semigeodesic charts exist in the case when the components of the affine connection are differentiable functions. The following is true:

Theorem 2. *For any affine connection determined by* $\Gamma^h_{ij}(x)? \in C^(U), r \geq 2$, *there exists a local transformation of coordinates determined by* $x' = f(x) \in C^r$ *such that the connection in the new coordinates* (U', x'), $U' \in U$, *satisfies* $\Gamma'^h_{11}(x') = 0$, $h = 1, \dots, n,$, *i.e. the coordinates* (U', x') *are pre-semigeodesic and the components* $\Gamma'^h_{ij}(x') \in C^{r-2}(U')$.

The differentiability class r is equal to $0, 1, 2, \ldots, \infty, \omega$, where $0, \infty$ and ω denote continuous, infinitely differentiable, and real analytic functions, respectively.

It therefore follows that the existence of a pre-semigeodesic chart is guaranteed in the case when the components of the affine connection ∇ are twice differentiable. The existence of this chart is not excluded in the case when the components are only continuous.

This is a joint work with Josef Mikeš.

[1] Dušek Z, Kowalski O. How many are affine connections with torsion. Arch. Math. 2014; 50(5): 257-264.

[2] Mikeš J, Vanžurová A, Hinterleitner I. Geodesic mappings and some generalizations. Palacky University Press, Olomouc. 2009.

[3] Mikeš J, Vanžurová A. Reconstruction of an affine connection in generalized Fermi coordinates. Bull. Malays. Math. Sci. Soc. 2016.