On Warped Product Manifolds Satisfying Ricci-Hessian Class Type Equations

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The m-Bakry-Emery-Ricci tensor Ric_f^m is an important object related to the study in Riemannian geometry, particularly in the study of Ricci flow and Ricci solitons. This tensor is defined by

$$Ric_f^m = Ric + Hessf - \frac{1}{m} df \otimes df; \qquad 0 < m \leqslant \infty$$

where f is a smooth function and m is a positive integer. When f is constant, the m-Bakry-Emery-Ricci tensor becomes the usual Ricci tensor so it gives an analog of the Ricci tensor for a Riemannian manifold. Moreover, m-Bakry-Emery-Ricci tensor arises from the warped product manifold $(M \times N, \tilde{g})$ endowed with the metric $\tilde{g} = g + e^{\frac{-2f}{m}} \bar{g}$.

In the present talk, we deal with a study of warped product manifold which is also a generalized quasi Einstein manifold. Then we give some characterizations about this manifold with related to the certain Ricci- Hessian type equations such as $Ric_f^m = \lambda g$, for some smooth function λ. Also, we obtain some rigidity conditions for this class of manifolds. Precisely, we prove that an m-generalized quasi Einstein manifold with a closed conformal vector field has a warped product structure of the form $I \times_{e^{q/2}} M^*$ where I is a real interval, M^* is an $(n-1)$-dimensional Riemannian manifold and q is a smooth function on I. Finally, we construct some non-trivial examples verifying our results.

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