The Proof of Blagojevic-Grujic-Zivaljevic Conjecture on Symmetric Products of Compact Riemann Surfaces with Punctures

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Let $M^2_{g,k}$ and $M^2_{g',k'}$ be compact Riemann surfaces with punctures ($g, g' \geq 0$ - genuses, $k, k' \geq 1$ - number of punctures). For any Hausdorff space X the quotient space $\mathrm{Sym}^n X := X^n/S_n$ is the n-th symmetric product of X, $n \geq 2$. It is well known, that $\mathrm{Sym}^n M^2_{g,k}$ is a smooth quasi-projective variety. Open manifolds $\mathrm{Sym}^n M^2_{g,k}$ and $\mathrm{Sym}^n M^2_{g',k'}$ are homotopy equivalent iff $2g + k = 2g' + k'$.

Blagojević-Grujić-Živaljević Conjecture (2003). Fix any $n \geq 2$, and two pairs (g, k) and (g', k') with the condition $2g+k = 2g'+k'$. If $g \neq g'$, then open manifolds $\mathrm{Sym}^n M^2_{g,k}$ and $\mathrm{Sym}^n M^2_{g',k'}$ are not continuously homeomorphic.

The conjecture was proved in 2003 by P. Blagojević, V. Grujić and R. Živaljević for the case $\max(g, g') \geq \frac{n}{2}$ (this implies the case $n = 2$). As far as the author knows, up to this moment there were no results if $\max(g, g') < \frac{n}{2}$.

The aim of this talk is to present the proof of the conjecture in full generality.