An Approximate Nerve Theorem

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In computational topology, an important role is played by the various versions of the Nerve Theorem, allowing us for example to conclude that the homology of a space is isomorphic to the homology of the nerve of its good cover, i.e. $H_*(X) \cong H_*(\mathcal{N}(\mathcal{U}))$.

However, these theorems do not account for the fact that in practical examples, measurements are often imprecise, so verifying that a cover is indeed good can only be done up to a certain precision. To solve this problem, we introduce the notion of an approximately good cover and show that an Approximate Nerve Theorem is valid for such covers. While the persistent homology of a filtered space and the persistent homology of the nerve of its ϵ-good cover need not be isomorphic, the theorem states that they are $2(D+1)\epsilon$-interleaved as persistence modules, where D is the dimension of the nerve. The proof relies on the properties of the Mayer-Vietoris spectral sequence.

This is a joint work with Primož Škraba.