

Two-sided multiplications and phantom line bundles

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Two-sided multiplications $M_{a,b} : x \mapsto axb$ on C^* -algebras A , where a and b are elements of A , are usually considered as basic building blocks for more general types of operators on A , as their finite sums (i.e. elementary operators) comprise both inner derivations and inner automorphisms. It is therefore natural to ask which operators $\phi : A \rightarrow A$ can be obtained as operator-norm limits of TMs.

Let us denote by $\text{TM}(A)$ the set of all TMs on A . We first show that $\text{TM}(A)$ is closed in the operator norm for all prime C^* -algebras A . On the other hand, if $A \cong \Gamma_0(\mathcal{E})$ is an n -homogeneous C^* -algebra, where \mathcal{E} is the canonical \mathbb{M}_n -bundle over the primitive spectrum X of A , we show that $\text{TM}(A)$ fails to be closed in the operator norm if and only if there exists a σ -compact open subset U of X and a phantom complex line subbundle \mathcal{L} of \mathcal{E} over U (i.e. \mathcal{L} is not globally trivial, but is trivial on all compact subsets of U). This phenomenon occurs whenever A is non-commutative and X is a CW-complex (or a topological manifold) of dimension $3 \leq d < \infty$.

This is a joint work with Richard M. Timoney (Trinity College Dublin).
