Transformations between Singer-Thorpe bases in 4-dimensional Einstein manifolds

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At each point of a 4-dimensional Einstein Riemannian manifold (M, g), the tangent space admits at least one so-called Singer-Thorpe basis (ST basis) with respect to the curvature tensor R at p. In this basis, up to standard symmetries and antisymmetries, just 5 components of the curvature tensor R are nonzero. For the space of constant curvature, the group O(4) acts as a transformation group between ST bases at T_pM and for the so-called 2-stein curvature tensors, the group $\mathrm{Sp}(1) \subset \mathrm{SO}(4)$ acts as a transformation group between ST bases. K. Sekigawa put the question "how many" Singer-Thorpe bases exist for a fixed curvature tensor R.

We give the complete list of Lie subgroups of SO(4) which act as transformation groups between ST bases for certain classes of Einstein curvature tensors. Special representations of groups SO(2), T^2, Sp(1) or U(2) are obtained. Further, we determine the so-called "universal Singer-Thorpe group", which is a finite group with 2304 elements and which transforms arbitrary Einstein curvature tensor into another ST basis.

We conjecture that the above groups give the complete answer to the Sekigawa problem. Part of the talk is the joint work with O. Kowalski.

[1] Dušek Z. Singer-Thorpe bases for special Einstein curvature tensors in dimension 4. Czechoslovak Mathematical Journal. to appear.

[2] Dušek, Z, Kowalski, O. Transformations between Singer-Thorpe bases in 4-dimensional Einstein manifolds. Hokkaido Mathematical Journal. to appear.