

# Algebro-geometric approach to Painlevé VI equation, Schlesinger systems, and Poncelet Polygons

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In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI  $(1/8, -1/8, 1/8, 3/8)$  equation starting from any closed Poncelet trajectory inscribed in a conic and circumscribed about another conic. We relate Hitchin's construction to an Okamoto transformation between Picard's solution and the general solution of the Painlevé VI  $(1/8, -1/8, 1/8, 3/8)$  equation. Moreover, we show that this Okamoto transformation can be presented in an invariant way, in terms of an Abelian differential of the third kind on an elliptic curve. The last observation allows us to obtain solutions to the corresponding Schlesinger system in terms of this differential as well. The solution to the Schlesinger system admits natural generalizations to higher genera. They appear to be related to higher-dimensional Poncelet polygons, closing the loop with the initial Hitchin's remarkable observation.

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