On classification problems in theory of differential equations: algebra + geometry

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In the present work we suggest a new approach to study classification problems in the theory of differential equations. Most of such problems are studied with the help of differential invariants, which are the differential analog of classical polynomial invariants. The dependencies between differential invariants and their invariant derivations locally define the equivalent class of differential equations with respect to the action of a given transformation group. But in the general case it is impossible to calculate these dependencies and hence, obtain an effective criterion for the equivalence problem.

To overcome this difficulty we introduce the algebraic structure in classification problem. Namely, we consider differential equations with algebraic coefficients. Then the dependencies between differential invariants for such equations are polynomials, and they can be calculated with the help of the computer.

This approach will be realized for the so–called Lie problem of classification of second order differential equations $y'' = F(x, y)$. We provide the geometric classification for the smooth right parts and algebraic classification for the rational right parts of such equations. We also discuss the generalizations of these results for the differential equations on the Riemann surfaces.