XIX Geometrical Seminar

Zlatibor, August 28 – September 4, 2016

BOOK OF ABSTRACTS

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The XIX Geometrical Seminar is dedicated to the memory of Professor Mileva Prvanović

Contents

Transversal hypersurfaces in weighted manifolds

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In this work we establish a relation between the transversality of two given hypersurfaces and the ellipticity of the weighted Newton transformations. In the second part we derive a flux formula of the Weighted Newtons transformations which can help us to minimize the weighted mean curvature by the geometry of the boundary of the hypersurface.

On evolution of positively curved invariant Riemannian metrics on special Wallach spaces

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In the papers [2] and [3], the authors studied the normalized Ricci flow equation on one special class of Riemannian manifolds called generalized Wallach spaces (or three-locally-symmetric spaces in other terms) according to the definitions of [5] and [7]. Note that the complete classification of generalized Wallach spaces is obtained recently (independently) in the papers [4] and [6]. A given generalized Wallach space can be determined by special parameters $a_i \in (0, 1/2]$ (see details in [5]). Our main result is the following

Theorem 1. *On a generalized Wallach space with $a_1 = a_2 = a_3 := a = 1/4$ the volume normalized Ricci flow evolves all generic invariant Riemannian metrics into metrics with positive Ricci curvature.*

It should be noted that the case $a \in (0, 1/2] \setminus \{1/4\}$ was studied in [1].

Acknowledgements. The author is indebted to Prof. Yu. Nikonorov for helpful discussions. The project was supported by Grant 1452/GF4 of MES of the Republic of Kazakhstan for 2015-2017.

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On Lorentzian spaces of constant sectional curvature

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We investigate Osserman-like conditions for Lorentzian curvature tensors that implies a constant sectional curvature. It is known that Osserman (moreover Zwei-stein) Lorentzian manifolds has a constant sectional curvature. We prove that some generalization of the Rakić duality principle implies a constant sectional curvature.

Toric origami manifolds and asymptotical properties of planar graphs

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Toric origami manifold is a generalization of symplectic toric manifold: instead of a symplectic form we allow close 2-forms which may degenerate in a nice way. Symplectic toric manifolds are classified by Delzant polytopes: the images of their moment maps. Similarly, origami toric manifolds are classified by “origami templates” (collections of Delzant polytopes with some folding data).

Masuda and Park proved that every 4-dimensional quasitoric manifold admits toric origami structure. The same question about higher dimensions arises. Using metric and coloring properties of planar graphs we proved the following fact: almost every simple 3-polytope supports a quasitoric manifold which is not toric origami.

The geometry of left-invariant structures on nilpotent Lie groups

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In this talk we continue studying invariant structures on homogeneous manifolds and their relations to the generalized Hermitian geometry. More exactly, we concentrate on left-invariant metric f-structures on special classes of nilpotent Lie groups. First, using the theory of canonical structures on homogeneous k- symmetric spaces [1], we construct nearly Kaähler and Hermitian f-structures on the 5-dimensional matrix Heisenberg group as well as on the 6-dimensional generalized Heisenberg group [2, 3]. It turns out that the method could be extended to some special matrix Lie groups which generalize the classical high-dimensional matrix Heisenberg groups. Further, some general results for 2-step nilpotent and other Lie groups were obtained. Finally, we dwell on the class of filiform Lie groups which also can be considered as a generalization (in some sense) of the classical 3-dimensional Heisenberg group. Specifically, many examples of left-invariant Hermitian f-structures on 6-dimensional filiform Lie groups were presented [4].

This is a joint work with Pavel Dubovik.

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On almost contact metric hypersurfaces in special Hermitian manifolds

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1. The classification of the almost Hermitian structures on first order differential-geometrical invariants can be rightfully attributed to the most significant results obtained by the outstanding American mathematician Alfred Gray and his Spanish colleague Luis M. Hervella. According to this classification, all the almost Hermitian structures are divided into 16 classes. Analytical criteria for each concrete structure to belong to one or another class have been obtained [6].

The class of special Hermitian manifolds (or W_3-manifolds, using Gray–Hervella notation) has been studied not so detailed as other so-called "small" Gray–Hervella classes of almost Hermitian manifolds. Some dozens of significant works are devoted to the nearly Kählerian, almost Kählerian and locally conformal Kählerian manifolds, but much less of articles are written about special Hermitian manifolds.

We remark also that the present work is a continuation of researches of the authors in the area of Hermitian manifolds, mainly six-dimensional (see, for example, [1–5] and others).

2. As it is known, an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J . Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [7].

We recall that the fundamental (or Kählerian) form of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

An almost Hermitian manifold is called Hermitian, if its structure is integrable. The following identity characterizes the Hermitian structure [6, 7]:

$$\nabla_X(F)(Y,Z) - \nabla_{JX}(F)(JY,Z) = 0,$$

where $X, Y, Z \in \aleph(M^{2n})$. A special Hermitian structure in addition must comply with the condition $\delta F = 0$, where δ is the codifferentiation operator [7].

3. The main results are the following:

1) The Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface in a special Hermitian manifold are obtained;

2) The Cartan structural equations of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on an oriented hypersurface in a special Hermitian manifold are selected;

3) A characterization in terms of the type number of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on hypersurfaces in special Hermitian manifolds is obtained;

4) A criterion of the minimality of such hypersurfaces in the terms of their type number is established.

These results are detailed for six-dimensional planar submanifolds [4, 5] of Cayley algebra that carry special Hermitian structures.

This is a joint work with G.A. Banaru.

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Quasitoric manifolds and small covers over neighborly polytopes

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We study small covers and quasitoric manifolds over duals of neighborly simplicial polytopes in dimension 4, 5 and 6. The explicit new examples of toric manifolds are constructed and the lifting conjectures is verified to be true.

Surface family with a common natural geodesic lift of a timelike curve in Minkowski 3-space

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In the present paper, we find a surface family possessing the natural lift of a given timelike curve as a geodesic in Minkowski 3-space. We express necessary and sufficient conditions for the given curve such that its natural lift is a geodesic on any member of the surface family. Finally, we illustrate the method with some examples.

Para H-projective transformations

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The present work is the contribution of Acad. Mileva Prvanovic (as a part of a joint project with C.L. Bejan and S.L. Druta-Romaniuc), elaborated in 2016.

Let M be a para-complex manifold, i.e. a differentiable manifold endowed with a para- complex structure P, which is parallel with respect to an affine connection ∇. Another connection with the same properties is called H-projectively related to ∇ if it has the same system of H-flat paths. A characterization of the H-projectively transformations are given here. Then a H- projective curvature tensor field HP is constructed, as an invariant under the H-projective transformations. When moreover, the manifold is endowed with a (semi-)Riemannian tensor field whose Levi-Civita connection is ∇, then this invariant HP is studied for two special cases: the locally decomposable Riemannian manifolds and the hyperbolic Kähler manifolds. The vanishing of HP is characterized in both cases.

Lagrangian submanifolds with constant angle functions in the nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$

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In this work, we discuss Lagrangian submanifolds in the nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$. First, we investigate how changing the immersion affects the so-called angle functions which describe the geometry of the submanifolds. Then, we classify the Lagrangian submanifolds in the nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$ for which all angle functions are constant. This classification is related to totally geodesic Lagrangian submanifolds or Lagrangian submanifolds with constant sectional curvature which are studied in [1] and [2], respectively. Finally, we focus on some specific values of angle functions.

Joint work with Marilena Moruz, Joeri Van der Veken and Luc Vrancken.

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Integrality of NIM-reps

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A NIM-rep is a non-negative integer matrix representation of a fusion ring. Fusion rings are found in many areas of mathematics, most notably in conformal field theory, and can be associated with affine Lie algebras. In this talk, we will investigate the integrality properties of affine algebra NIM-reps.

On classification problems in theory of differential equations: algebra + geometry

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In the present work we suggest a new approach to study classification problems in the theory of differential equations. Most of such problems are studied with the help of differential invariants, which are the differential analog of classical polynomial invariants. The dependencies between differential invariants and their invariant derivations locally define the equivalent class of differential equations with respect to the action of a given transformation group. But in the general case it is impossible to calculate these dependencies and hence, obtain an effective criterion for the equivalence problem.

To overcome this difficulty we introduce the algebraic structure in classification problem. Namely, we consider differential equations with algebraic coefficients. Then the dependencies between differential invariants for such equations are polynomials, and they can be calculated with the help of the computer.

This approach will be realized for the so–called Lie problem of classification of second order differential equations $y'' = F(x, y)$. We provide the geometric classification for the smooth right parts and algebraic classification for the rational right parts of such equations. We also discuss the generalizations of these results for the differential equations on the Riemannian surfaces.

Distribution of flows on edges of a hypercube

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A signal on edge of a hypercube flows and divides at incoming vertex. A combinatorial problem of distribution of the signal on vertices is considered.

Half conformally flat gradient Ricci almost solitons in higher signature

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A *gradient Ricci soliton* is a triple (M, g, f) where (M, g) is a pseudo-Riemannian manifold and f is a function satisfying the Ricci soliton equation

$$\mathrm{Hes}_f + \rho = \lambda g,$$

where Hes_f denotes the Hessian of f, ρ denotes de Ricci tensor and λ is a real number. The main interest of gradient Ricci solitons comes from the fact that they correspond to self-similar solutions of the Ricci flow $\partial_t g(t) = -2\rho_{g(t)}$. The gradient Ricci soliton equation can be generalized letting λ be a function on M and then (M, g, f) is said to be a *gradient Ricci almost soliton*. Some gradient Ricci almost solitons with non-constant λ correspond to self-similar solutions of some geometric flows. The *Ricci-Bourguignon flow* is given by the equation $\frac{\partial}{\partial t} g(t) = -2(\rho(t) - \kappa\tau(t)\, g(t))$, (where $\kappa \in \mathbb{R}$ and τ denotes the scalar curvature). The self-similar solutions of this flow are called *κ-Einstein solitons* and correspond to the equation

$$\mathrm{Hes}_f + \rho = (\kappa\,\tau + \mu)\, g$$

for some $\kappa, \mu \in \mathbb{R}$. Hence, they are a special family of gradient Ricci almost solitons with soliton function $\lambda = \kappa\,\tau + \mu$ (see [3]).

In this talk we will describe the local structure of half conformally flat gradient Ricci solitons and gradient Ricci almost solitons in dimension four with special attention to higher signature. We will see that they are locally conformally flat in a neighbourhood of any point where the gradient of the potential function is non-null, as it occurs in the Riemannian setting [4]. Moreover, in Lorentzian signature the half conformally flat condition is equivalent to local conformal flatness. In opposition, if the gradient of the potential function is null, then the soliton is locally isometric to the cotangent bundle of an affine surface equipped with the Riemannian extension of the connection [1], whereas the almost soliton is a steady traceless κ-Einstein soliton which is also realized on the cotangent bundle of an affine surface [2]. In both cases, a method to construct half conformally flat examples which are not locally conformally flat is given. The situation described in neutral signature, both for gradient Ricci solitons and gradient Ricci almost solitons, contrast with the Riemannian one.

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The computer modeling of gluing flat images algorithms

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In treatise one of the important tasks of modern computer geometry is being considered. The main idea of this task is to create gluing flat images algorithms of the same object in space. We can get image data with the help of central projection from different points of space. We construct a numerical simulation for each of the algorithms - a simple linear, normalized linear and direct. The stability of projective transformation to a perturbation of the initial data is being estimated. The accuracy and speed of the algorithms are being calculated.

The results confirm the hypothesis of G.V. Nosovskiy and E.S. Skripka that their proposed direct algorithm is the most stable to perturbation coordinates of conjugate points, as well as to changes in their configuration.

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The inverse along an element in semigroups and rings

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The concept of the inverse along an element was introduced by X. Mary in 2011. Later H.H.Zhu etc. introduced the one-sided inverse along an element. In this talk, we first give a new existence criterion for the one-sided inverse along a product and characterize the existence of Moore-Penrose inverse by means of one-sided invertibility of certain element in a ring. Then we get some characterizations where a^*a (or aa^*) is invertible along a. Finally, we consider the conditions when a is invertible along aa^* (or a^*a).

Generalized inverses and some problems of completions of operator matrices

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We will address some recent results on generalized inverses and certain problems of completions of operator matrices. We will show interesting connections between these two, at first sight totally unrelated, branches of research in mathematics.

Fixed point theorems of Perov type and Ulam's stability

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Perov theorem claims existence and uniqueness of a fixed point for a new kind of a contraction on a generalized metric space involving positive matrix converging to zero. We give extensions of this result in various settings and include different types of contractions. The contractive condition of Perov type on cone metric spaces is defined and it involves an operator instead of a matrix. We also discuss Ulam-Hyers stability of functional equations, making a new approach by applying fixed point results of Perov type.

Asymptotic stability of combination of contact waves with rarefaction waves for one-dimensional radiative hydrodynamic systems

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We investigate the Cauchy problem of one-dimensional radiative hydrodynamic systems. We construct the solution of vicious combination of contact waves with rarefaction waves and obtain its asymptotic stability for the Cauchy problem by energy estimations.

This is a joint work with Lin Chang.

On generalized invertibility of operators

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In this talk, we will first report some recent results on the general invertibility of the products and differences of idempotents and generalized projections. The invertibility, the group invertibility and the k-potency of the linear combinations of idempotents are investigated. Second we will give some common characterizations and various individual properties of the star ordering, the left star ordering, the right star ordering and the minus partial ordering of bounded operators on a Hilbert space. The several properties for which the common star lower or star upper bound exists regarding the relationships among operators and projections are given. The much simpler matrix representations with respect to star order relation are obtained.

Curvature properties of some class of warped product manifolds

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Warped product manifolds of dimension $n \geq 4$, with p-dimensional base, $p = 1, 2$, satisfy some pseudosymmetry type curvature conditions. These conditions are formed from the metric tensor g, the Riemann-Christoffel curvature tensor R, the Ricci tensor S and the Weyl conformal curvature C of the considered manifolds. In particular, if $p = 2$ and the fiber is a semi-Riemannian space of constant curvature (when $n \geq 5$) then the $(0, 6)$-tensors $R \cdot R - Q(S, R)$ and $C \cdot C$ of such warped products are proportional to the $(0, 6)$-tensor $Q(g, C)$ and the tensor C is a linear combination of some Kulkarni-Nomizu products formed from the tensors g and S. We also present curvature

properties of this kind of quasi-Einstein and 2-quasi-Einstein manifolds, and in particular, of the Gödel metric, generalized spherically symmetric metrics and generalized Vaidya metrics. Our talk bases on [1–6].

Joint work with Małgorzata Głogowska and Jan Jełowicki (Wrocław).

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Nonsingular Big Bang solutions in nonlocal modified gravity

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After discovery of accelerating expansion of the Universe, there has been a renewed interest in gravity modification. One of promising approaches is nonlocal modification with the scalar curvature R in the action replaced by a suitable function $\mathcal{F}(R, \Box)$, where $\Box$ is the Laplace-Beltrami operator. In particular we analyze the modification in the form

$$S = \int \left(\frac{R - 2\Lambda}{16\pi G} + R^p \mathcal{F}(\Box) R^q \right) \sqrt{-g} \mathrm{d}^4 x$$

where $\mathcal{F}(\Box)$ is an analytic function. We present a few $a(t)$ nonsingular bounce cosmological solutions for the above action using $FLRW$ metric.

This is joint work with Branko Dragovich, Zoran Rakić and Jelena Stanković.

Some class of Kenmotsu manifolds with generalized Tanaka-Webster connection

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The talk aims at discussing a generalized Tanaka-Webster connection on a Kenmotsu manifold. We study the conharmonic curvature tensor with respect to the generalized Tanaka- Webster connection $\widetilde{\nabla}$ and also characterize conharmonically flat and locally ϕ- conharmonically symmetric Kenmotsu manifold with respect to the connection $\widetilde{\nabla}$. Besides these we also classify Kenmotsu manifolds which satisfy $\widetilde{K} \cdot \widetilde{R} = 0$ and $\widetilde{P} \cdot \widetilde{K} = 0$, where $\widetilde{K}$ and $\widetilde{P}$ are the conharmonic curvature tensor, the projective curvature tensor and Riemannian curvature tensor, respectively with respect to the connection $\widetilde{\nabla}$.

STCR-lightlike submanifolds of an indefinite Kähler manifold

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In the present paper, we introduce a new class of lightlike submanifolds, namely, Screen Transversal Cauchy Riemann (STCR)-lightlike submanifolds, of indefinite Kähler manifolds. We show that this new class is an umbrella of screen transversal lightlike, screen transversal totally real lightlike and CR-lightlike submanifolds. We give an example of a STCR lightlike submanifold, investigate the integrability of various distributions obtain a characterization of such lightlike submanifolds in a complex space form and find new conditions for the induced connection to be a metric connection.

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Ultrametric spaces and their applications

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In ultrametric (non-Archimedean) spaces, distances satisfy strong triangle inequality, $d(x,y) \leq max\{d(x,z), d(y,z)\}$, instead of the usual one, $d(x,y) \leq d(x,z) + d(y,z)$. From the point of view of Archimedean geometry, ultrametric spaces have some very unusual properties, which turn out to be natural in many applications. There are many examples of ultrametric spaces in mathematics, physics, biology, linguistics and information sciences. Ultrametricity [1] is a particularly appropriate mathematical tool to model and describe the hierarchy and similarity within bioinformation systems. The most advanced examples of ultrametrics are related to p-adic numbers [2] and their applications [3].

We will present a review of ultrametric spaces and their applications, in particular in the genetic code [4] and bioinformation systems.

This is joint work with Nataša Ž. Mišić .

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Algebro-geometric approach to Painleve VI equation, Schlesinger systems, and Poncelet Polygons

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In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI (1/8, -1/8, 1/8, 3/8) equation starting from any closed Poncelet trajectory inscribed in a conic and circumscribed about another conic. We relate Hitchinś construction to an Okamoto transformation between Picardś solution and the general solution of the Painlevé VI (1/8, -1/8, 1/8, 3/8) equation. Moreover, we show that this Okamoto transformation can be presented in an invariant way, in terms of an Abelian differential of the third kind on an elliptic curve. The last observation allows us to obtain solutions to the corresponding Schlesinger system in terms of this differential as well. The solution to the Schlesinger system admits natural generalizations to higher genera. They appear to be related to higher-dimensional Poncelet polygons, closing the loop with the initial Hitchin's remarkable observation.

This is a joint work with V. Shramchenko.

The research has been partially supported by the project 174020 of the Serbian Ministry for Education, Science, and Technologies, the NSF grant 1444147, and the University of Texas at Dallas.

Transformations between Singer-Thorpe bases in 4-dimensional Einstein manifolds

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At each point of a 4-dimensional Einstein Riemannian manifold (M, g), the tangent space admits at least one so-called Singer-Thorpe basis (ST basis) with respect to the curvature tensor R at p. In this basis, up to standard symmetries and antisymmetries, just 5 components of the curvature tensor R are nonzero. For the space of constant curvature, the group O(4) acts as a transformation group between ST bases at T_pM and for the so-called 2-stein curvature tensors, the group Sp(1) $\subset$ SO(4) acts as a transformation group between ST bases. K. Sekigawa put the question "how many" Singer-Thorpe bases exist for a fixed curvature tensor R.

We give the complete list of Lie subgroups of SO(4) which act as transformation groups between ST bases for certain classes of Einstein curvature tensors. Special representations of groups SO(2), T^2, Sp(1) or U(2) are obtained. Further, we determine the so-called "universal Singer-Thorpe group", which is a finite group with 2304 elements and which transforms arbitrary Einstein curvature tensor into another ST basis.

We conjecture that the above groups give the complete answer to the Sekigawa problem. Part of the talk is the joint work with O. Kowalski.

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Sharp inequalities involving the Ricci curvature for Riemannian submersions

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In this paper, we obtain sharp inequalities on Riemannian manifolds admitting a Riemannian submersion and give some characterizations using these inequalities. We improve Chen-Ricci inequality for Riemannian submersion and present some examples which satisfy this inequality.

A Hamiltonian approach to Thermodynamics

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In the present work we develop a strictly Hamiltonian approach to Thermodynamics. A thermodynamic description based on symplectic geometry is introduced, where all thermodynamic processes can be described within the framework of Analytic Mechanics. Our proposal is constructed on top of a usual symplectic manifold, where phase space is even dimensional and one has well-defined Poisson brackets. The main idea is the introduction of an extended phase space where thermodynamic equations of state are realized as constraints. We are then able to apply the canonical transformation toolkit to thermodynamic problems. Throughout this development, DiracŠs theory of constrained systems is extensively used. To illustrate the formalism, we consider paradigmatic examples, namely, the ideal, van der Waals and Clausius gases.

Precesion of four-dimensional rigid body motion

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For the four-dimensional Lagrange top on $e(4)$ a particular solution is constructed. Given solution represents a four-dimensional regular precession. Starting from it, a four-dimensional analogue of the Grioli nonvertical regular precession of an asymmetric heavy rigid body is constructed.

This is a joint work with Vladimir Dragović.

On the structure of gradient Ricci solitons with constant scalar curvature

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The purpose of this talk is to review some recent results on the geometry of gradient Ricci solitons with constant scalar curvature. Rigid Ricci solitons have constant scalar curvature, but no other non-trivial examples are known. Conditions on the number of different Ricci curvatures as well as on the rank of the Ricci operator suffice to show that the constancy of the scalar curvature leads to rigidity. Finally, a structure result shows that any gradient Ricci soliton with constant scalar curvature is a vector bundle over a totally geodesic submanifold.

Hypersurfaces in space forms satisfying some curvature conditions

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We present curvature properties of some class of hypersurfaces isometrically immersed in spaces of constant curvature satisfying some curvature conditions of pseudosymmetry type. Our talk bases on [1] and [2].

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Double rotational surfaces in Euclidean 4-space

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A logical continuation of our previous study of twisted surfaces (see [2] and the references therein), is to generalize this concept to a higher dimensional space. Hence, in this talk, I want to present some first results on double rotational surfaces in 4-dimensional Euclidean space.

Since there are two possible rotations in Euclidean 4-space, namely rotations about a plane and rotations about a point, one has to consider all possible combinations of the two simultaneous rotations that are used to construct a double rotational surface. When starting with a planar curve as profile curve, this leads to only one possible parametrization of a double rotational surface, up to a transformation. In the other cases, either a twisted surface in Euclidean 3-space or, (a part of) a plane is obtained. The double rotational surface of course incorporates the ordinary rotational surface in Euclidean 4-space as a special case.

For these double rotational surfaces some curvature properties are examined. That is, flat double rotational surfaces and minimal double rotational surfaces are studied. Imposing these curvature conditions on a double rotational surface, leads to large expressions which can be studied systematically using for instance Maple. As to be expected, the results incorporate several new surfaces among which cones defined over a 4-dimensional version of Clelia curves (for an overview of properties of Clelia curves in 3-space, see [1] and the references therein).

In the light of these results it is of course promising to transfer this study to Minkowski 4-space as further research. One expects to find, due to the different causal character of vectors existing in that space, even more interesting new surfaces.

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Two-sided multiplications and phantom line bundles

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Two-sided multiplications $M_{a,b} : x \mapsto axb$ on C^*-algebras A, where a and b are elements of A, are usually considered as basic building blocks for more general types of operators on A, as their finite sums (i.e. elementary operators) comprise both inner derivations and inner automorphisms. It is therefore natural to ask which operators $\phi : A \to A$ can be obtained as operator-norm limits of TMs.

Let us denote by $\mathrm{TM}(A)$ the set of all TMs on A. We first show that $\mathrm{TM}(A)$ is closed in the operator norm for all prime C^*-algebras A. On the other hand, if $A \cong \Gamma_0(\mathcal{E})$ is an n-homogeneous C^*- algebra, where $\mathcal{E}$ is the canonical $\mathbb{M}_n$-bundle over the primitive spectrum X of A, we show that $\mathrm{TM}(A)$ fails to be closed in the operator norm if and only if there exists a σ-compact open subset U of X and a phantom complex line subbundle $\mathcal{L}$ of $\mathcal{E}$ over U (i.e. $\mathcal{L}$ is not globally trivial, but is trivial on all compact subsets of U). This phenomenon occurs whenever A is non-commutative and X is a CW-complex (or a topological manifold) of dimension $3 \leq d < \infty$.

This is a joint work with Richard M. Timoney (Trinity College Dublin).

A 15-vertex triangulation of the quaternionic projective plane

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In 1992, Brehm and Kühnel constructed a 8-dimensional simplicial complex M^8_{15} with 15 vertices as a candidate to be a minimal triangulation of the quaternionic projective plane. They managed to prove that it is a manifold "like a projective plane" in the sense of Eells and Kuiper, ie a manifold that admits a Morse function with exactly 3 critical points. However, it was not known until now if this complex is PL homeomorphic (or at least homeomorphic) to $\mathbb{H}P^2$. This problem was reduced to the computation of the first rational Pontryagin class of this combinatorial manifold. Realizing an algorithm due to Gaifullin, we compute the first Pontryagin class of M^8_{15}. As a result, we obtain that it is indeed a minimal triangulation of $\mathbb{H}P^2$.

An approximate nerve theorem

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In computational topology, an important role is played by the various versions of the Nerve Theorem, allowing us for example to conclude that the homology of a space is isomorphic to the homology of the nerve of its good cover, i.e. $H_*(X) \cong H_*(\mathcal{N}(\mathcal{U}))$.

However, these theorems do not account for the fact that in practical examples, measurements are often imprecise, so verifying that a cover is indeed good can only be done up to a certain precision. To solve this problem, we introduce the notion of an approximately good cover and show that an Approximate Nerve Theorem is valid for such covers. While the persistent homology of a filtered space and the persistent homology of the nerve of its ϵ-good cover need not be isomorphic, the theorem states that they are $2(D+1)\epsilon$-interleaved as persistence modules, where D is the dimension of the nerve. The proof relies on the properties of the Mayer-Vietoris spectral sequence.

This is a joint work with Primož Škraba.

Quasisymmetric function invariant of graphs

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We introduce a new quasisymmetric function invariant of graphs obtained as the enumerator function of lattice points in the normal fan of graph-associahedra and show some of its combinatorial properties.

The proof of Blagojevic-Grujic-Zivaljevic conjecture on symmetric products of compact Riemann surfaces with punctures

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Let $M^2_{g,k}$ and $M^2_{g',k'}$ be compact Riemann surfaces with punctures ($g, g' \geq 0$ - genuses, $k, k' \geq 1$ - number of punctures). For any Hausdorff space X the quotient space $\mathrm{Sym}^n X := X^n/S_n$ is the n-th symmetric product of X, $n \geq 2$. It is well known, that $\mathrm{Sym}^n M^2_{g,k}$ is a smooth quasi-projective variety. Open manifolds $\mathrm{Sym}^n M^2_{g,k}$ and $\mathrm{Sym}^n M^2_{g',k'}$ are homotopy equivalent iff $2g + k = 2g' + k'$.

Blagojević-Grujić-Živaljević Conjecture (2003). Fix any $n \geq 2$, and two pairs (g, k) and (g', k') with the condition $2g+k = 2g'+k'$. If $g \neq g'$, then open manifolds $\mathrm{Sym}^n M^2_{g,k}$ and $\mathrm{Sym}^n M^2_{g',k'}$ are not continuously homeomorphic.

The conjecture was proved in 2003 by P. Blagojević, V. Grujić and R. Živaljević for the case $\max(g, g') \geq \frac{n}{2}$ (this implies the case $n = 2$). As far as the author knows, up to this moment there were no results if $\max(g, g') < \frac{n}{2}$.

The aim of this talk is to present the proof of the conjecture in full generality.

Some characterizations on submanifolds of Riemannian and Kählerian product manifolds

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In the present paper, submanifolds of Riemannian and Kaehlerian product manifolds are investigated. Some properties of an orthonormal basis in proper slant submanifolds of a Riemannian product manifold are mentioned. Some characterizations are obtained for proper slant submanifolds using the properties this orthonormal basis.

On warped product manifolds satisfying Ricci-Hessian class type equations

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The m-Bakry-Emery-Ricci tensor Ric_f^m is an important object related to the study in Riemannian geometry, particularly in the study of Ricci flow and Ricci solitons. This tensor is defined by

$$Ric_f^m = Ric + Hessf - \frac{1}{m} df \otimes df; \quad 0 < m \leqslant \infty$$

where f is a smooth function and m is a positive integer. When f is constant, the m-Bakry-Emery-Ricci tensor becomes the usual Ricci tensor so it gives an analog of the Ricci tensor for a Riemannian manifold. Moreover, m-Bakry-Emery-Ricci tensor arises from the warped product manifold $(M \times N, \tilde{g})$ endowed with the metric $\tilde{g} = g + e^{\frac{-2f}{m}} \bar{g}$.

In the present talk, we deal with a study of warped product manifold which is also a generalized quasi Einstein manifold. Then we give some characterizations about this manifold with related to the certain Ricci-Hessian type equations such as $Ric_f^m = \lambda g$, for some smooth function λ. Also, we obtain some rigidity conditions for this class of manifolds. Precisely, we prove that an m-generalized quasi Einstein manifold with a closed conformal vector field has a warped product structure of the form $I \times_{e^{q/2}} M^*$ where I is a real interval, M^* is an $(n-1)$-dimensional Riemannian manifold and q is a smooth function on I. Finally, we construct some non-trivial examples verifying our results.

This is joint work with Sezgin Altay Demirbağ.

Sectional curvature in 4-dimensional manifolds

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This talk will review the known work on the study of the sectional curvature function on 4-dimensional manifolds for metrics with positive definite or Lorentz signature. It will then deal with the remaining case of neutral signature. The talk will be divided into four parts; first, some historical comments of the sectional curvature function, second, a study of the structure of 2-spaces of the tangent space for neutral signature and a resume of the algebraic structure of the Weyl conformal tensor which turns out to be useful in this study, third, the study of the actual sectional curvature function for this case and finally a discussion of the degree of uniqueness of the metric from which a given sectional curvature function came.

On the existence of pre-semigeodesic coordinates

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In the present lecture we consider the problem of the existence of pre-semigeodesic coordinates on manifolds with affine connection. We proved that pre-semigeodesic coordinates exist in the case when the components of the affine connection are differentiable functions.

Let $A_n = (M, \nabla)$ be an n-dimensional manifold M with the affine connection ∇, dimension $n \geq 2$, and let $U \subset M$ be a coordinate neighbourhood at the point $x_0 \in U$. A couple (U, x) is a coordinate map on A_n.

Semigeodesic coordinate systems on surfaces and (pseudo-) Riemannian manifolds are generalized in the following way [2]:

Definition 1. Coordinates (U, x) in A_n are called *pre-semigeodesic coordinates* if one system of coordinate lines is geodesic and the natural parameter is just the first coordinate.

In $[1, 2]$ the following theorems were proved.

Theorem 2. *The conditions* $\Gamma^h_{11}(x) = 0$, $h = 1, \dots, n$, *are satisfied in* (U, x) *if and only if* (U, x) *is pre-semigeodesic.*

We thought that the existence of this chart is trivial. This problem is obviously more difficult than we supposed. This was observed in [3] where precisely the existence of pre-semigeodesic charts was proved in the case when the components of the affine connection are real analytic functions.

We proved that the pre-semigeodesic charts exist in the case when the components of the affine connection are differentiable functions. The following is true:

Theorem 3. *For any affine connection determined by* $\Gamma^h_{ij}(x)? \in C^(U), r \geq 2$, *there exists a local transformation of coordinates determined by* $x' = f(x) \in C^r$ *such that the connection in the new coordinates* (U', x'), $U' \in U$, *satisfies* $\Gamma'^h_{11}(x') = 0$, $h = 1, \dots, n$,, *i.e. the coordinates* (U', x') *are pre-semigeodesic and the components* $\Gamma'^h_{ij}(x') \in C^{r-2}(U')$.

The differentiability class r is equal to $0, 1, 2, \ldots, \infty, \omega$, where $0, \infty$ and ω denote continuous, infinitely differentiable, and real analytic functions, respectively.

It therefore follows that the existence of a pre-semigeodesic chart is guaranteed in the case when the components of the affine connection ∇ are twice differentiable. The existence of this chart is not excluded in the case when the components are only continuous.

This is a joint work with Josef Mikeš.

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Common fixed points for pair of mappings

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This talk is devoted to $g-f$ quasi-contractions on a cone metric space. In what follows we shall first give the definition of the very notion itself and then present some fixed point results on $g-f$ quasi-contractions. Among other things, we will see how a recent result of H.L. Guang and Z. Xian can be generalized so that one of the main results of Ćirić is recovered from it.

Most of the presented results actually represent generalizations and unifications of fixed point theorems of Das and Naik, Ćirić, Jungck and Ume in terms of ω-distances on a complete metric space.

Chord diagrams: framings, operations and maps

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In the talk we consider chord diagrams, i.e. a circle with a set of chords. Chord diagrams are very useful in knot theory and in the theory of embedding of curves. It is well known that chord diagrams modulo 4T-relations admit a Hopf algebra structure, where the multiplication is given by a connected sum, but in the case of framed chord diagrams the situation is different. To show what happens in the case of framed chord diagrams we present invariants (modulo 4T-relations) and maps on framed chord diagrams. Also we touch the case of linear diagrams, i.e. instead of the circle we have a line.

Solution of the qc Yamabe equation on a 3-Sasakian manifold and extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg group

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A complete solution to the quaternionic contact Yamabe equation on the qc sphere of dimension $4n+3$ as well as on the quaternionic Heisenberg group is given. A uniqueness theorem for the qc Yamabe problem in a compact locally 3-Sasakian manifold is shown and the extremals of the Sobolev-Folland-Stein inequality on the quaternionoic Heisenberg group are confirmed.

Singular motion of a symmetric Manakov top

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We consider a motion of $SO(n_1) \times \cdots \times SO(n_p)$–symmetric free rigid body around a fixed point [6]. The system is integrable in a noncommutative sense on by means of the Manakov and the Noether integrals [1, 2]. The geodesic flow of the reduced submersion metric on the homogeneous space $SO(n)/SO(n_1) \times \cdots \times SO(n_p)$ is also integrable [1–3], giving an important family of homogeneous spaces with integrable geodesic flows.

Further, an interesting problem is the description of trajectories of the corresponding Euler equations on the subspace $\mathfrak{v}$ of $so(n)$ given by the zero value of the Noether integrals [4]. Some of the Manakov integrals become dependent, but the new polynomial integrals on $\mathfrak{v}$ appear. In the case of the $SO(n-2)$–symmetry, it is shown that almost all trajectories are periodic and that the motion can be expressed in terms of the elliptic functions. In the case of the $SO(n-3)$–symmetry, we prove the solvability of the problem by using Kozlov's result on the Euler-Jacobi-Lie theorem [5].

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The (b, c)-inverse of products and lower triangular matrices

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Let S be a semigroup and $b, c \in S$. The concept of (b, c)-inverses was introduced by Drazin in 2012. It is well known that the Moore-Penrose inverse, the Drazin inverse, the Bott-Duffin inverse, the inverse along an element, the core inverse and dual core inverse are all special cases of the (b, c)-inverse. In this paper, a new relationship between the (b, c)-inverse and the Bott-Duffin (e, f)-inverse is established. The relations between the (b, c)-inverse of paq and certain classes of generalized inverses of pa and aq, and the (b', c')-inverse of a are characterized for some $b', c' \in S$, where $p, a, q \in S$. Necessary and sufficient conditions for the existence of the (B, C)-inverse of a lower triangular matrix over an associative ring R are also given, and its expression is derived, where B, C are regular triangular matrices.

On the generalized constant ratio submanifolds in Minkowski spaces

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A submanifold M in Minkowski space is said to be a generalized constant ratio (GCR) if the tangential part of its position vector is one of its canonical principal direction. In this talk, first, we will present a short survey on GCR submanifolds in semi-Euclidean spaces. Then, we will give some of classification results that we have obtained recently.

Topology of isoenergy surfaces for Kovalevskaya integrable case on Lie algebra so(4)

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Let us consider the six-dimensional space $\mathbb{R}^6(\mathbf{J},\mathbf{x})$ and the following one- parameter family of Poisson brackets depending on the real parameter κ:

$$\{J_i, J_j\} = \varepsilon_{ijk}J_k, \quad \{J_i, x_j\} = \varepsilon_{ijk}x_k, \quad \{x_i, x_j\} = \kappa\varepsilon_{ijk}J_k.$$

These brackets have two Casimir functions:

$$f_1 = x_1^2 + x_2^2 + x_3^2 + \kappa(J_1^2 + J_2^2 + J_3^2), \qquad f_2 = x_1J_1 + x_2J_2 + x_3J_3.$$

I.V. Komarov in his paper [4] showed that the Kovalevskaya integrable case in rigid body dynamics can be included in a one-parameter family of integrable Hamiltonian systems on this pencil of Lie algebras so(4) − e(3) − so(3, 1). The Kovalevskaya top was realized as a system on Lie algebra e(3). The Hamiltonian H and first integral have the following form:

$$H = J_1^2 + J_2^2 + 2J_3^2 + 2c_1x_1,$$
$$K = (J_1^2 - J_2^2 - 2c_1x_1 + \kappa c_1^2)^2 + (2J_1J_2 - 2c_1x_2)^2,$$

where c_1 is an arbitrary constant.

In the case of $\varkappa > 0,\ a > 0$ the common level surfaces $M^4_{a,b} = f_1 = a, f_2 = b$ of Casimir functions are compact orbits of coadjoint representation and symplectic leaves of the Lie- Poisson bracket. Every regular $M^4_{a,b}$ has a structure of Liouville foliation. Every two-dimensional torus is a closure of trajectories of this system. The purpose of topological analysis is the calculation of Fomenko-Zieschang invariant for 3-dimensional isoenergy surfaces [1]. We continue I.K. Kozlov's research of this system. In [3] the bifurcation diagrams were constructed and nondegenerate critical points of the rank 0 were described. Some new results will be presented including the list of all types of isoenergy surfaces of the system.

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An investigation of symmetric and skew-symmetric recurrent tensors of second order on $4-$dimensional manifolds

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This work is in collaboration with Graham Hall. In it, we first investigate the recurrence structure of second order symmetric tensors on a $4-$dimensional manifold admitting a metric whose signature is $(+,+,-,-)$ (neutral signature). (The results for $(+,+,+,-)$ (Lorentz signature) and $(+,+,+,+)$ (positive definite signature) are known but will be reviewed). We present some basic concepts about $4-$dimensional manifolds with neutral signature, recurrence conditions and holonomy theory. Secondly, the problem of parallel (or scalable as to be parallel), second order symmetric tensor fields is considered by using techniques based on the classification of such tensors and holonomy theory. This idea is then enlarged to recurrent tensor fields which are not in this class and which will be called properly recurrent second order symmetric tensor fields. All possible Segre types and holonomy types are found for these problems and an application of this study is given. Finally, this investigation is extended to second order skew-symmetric tensor fields which are referred to bivectors. The proper recurrence for bivectors is examined on a $4-$dimensional manifold of Lorentz signature and some results related to them are given. Some remarks for this problem are also given when the metric has positive definite signature.

Some remarks to the theorem of Wan for HQC mappings

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We give a new approach to the theorem of Wan which is related to the hyperbolic bi-Lipschicity of the K-quasiconformal, $K \geqslant 1$, hyperbolic harmonic self diffeomorphisms of the unit disk $\mathbb{D}$. Especially, we will present new results where the codomains of those mappings are simply connected domains.

Geometries in non-local gravity

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I will review the achievements of the very promising for the moment explicit candidate for a renormalizable gravity theory.

Some systems of nonlinear PDE which are soluble in closed form

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The goal of this lecture is to study so-called Riemannian manifolds of conullity two. This means that, at any point, there is an orthonormal basis such that the each curvature component with at least three distinct indices is always equal to zero. Most "geometric classes" of such manifolds in dimension 3 can be expressed in an explicit form, using only arithmetic operations, differentiation and integration, involving some number of arbitrary functions.

The natural brackets on couples of vector fields and 1-forms

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This contribution is a joint work of M. Doupovec (Brno), J. Kurek (Lublin) and W. M. Mikulski (Krakow).

Let $\mathcal{M}f_m$ be the category of m-dimensional C^∞ manifolds and their embeddings. If $m \geq 2$, we classify all bilinear $\mathcal{M}f_m$-natural operators $A\colon (T \oplus T^*) \times (T \oplus T^*) \rightsquigarrow (T \oplus T^*)$ transforming $X^i \oplus \omega^i \in \mathcal{X}(M) \oplus \Omega^1(M)(i = 1, 2)$ for m-manifolds M into $A(X^1 \oplus \omega^1, X^2 \oplus \omega^2) \in \mathcal{X}(M) \oplus \Omega^1(M)$. Next, if $m \geq 2$, we find all bilinear $\mathcal{M}f_m$-natural brackets on $\mathcal{X}(M) \oplus \Omega^1(M)$ satisfying the Leibniz rule, and we find all $\mathcal{M}f_m$-natural Lie algebra brackets on $\mathcal{X}(M) \oplus \Omega^1(M)$.

Hochschild (co)homology of exterior algebras using algebraic Morse theory

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We compute the additive and multiplicative structure of $HH^*(A;A)$, where A is the n-th exterior algebra over a field. We provide concise presentations of algebras $HH_*(A;A)$ and $HH^*(A;A)$, as well as determine their generators in the Hochschild complex. Lastly, we compute an explicit free resolution (spanned by multisets) of the A^e-module A and describe the homotopy equivalence to its bar resolution. All this is done using Sköldberg's algebraic generalization of Forman's discrete Morse theory.

2-truncated cubes and their toric spaces

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To any convex simple n-dimensional polytope P with m facets one can associate its moment-angle manifold $\mathcal{Z}_P$ – one of the main objects of study in toric topology. It was introduced firstly by M.Davis and T. Januszkiewicz as a generalization of the notions of a quasitoric manifold and a projective toric manifold. V. Buchstaber and T. Panov proved that $\mathcal{Z}_P$ is a smooth $(m+n)$-dimensional closed 2-connected manifold with a compact torus T^m action, whose orbit space is homeomorphic to the polytope P itself. The topology of $\mathcal{Z}_P$ is governed by the face lattice of P and can be very complicated.

In our talk we shall introduce several equivalent definitions of $\mathcal{Z}_P$ arising in toric and symplectic geometry, their relation to smooth toric varieties, and then discuss additive structure and Massey products in cohomology of moment-angle manifolds $\mathcal{Z}_P$ when P is a 2-truncated cube, that is a consecutive cut of only codimension 2 faces starting with a cube. V. Buchstaber and V. Volodin showed that any flag nestohedron can be realized as a 2-truncated cube and proved Gal's conjecture on γ-vectors for them. We introduce a family of n-dimensional 2-truncated cubes P, such that there is a nontrivial n-fold Massey product in cohomology of the moment-angle manifold $\mathcal{Z}_P$ for any $n \geq 2$ and present our family of 2-truncated cubes as flag nestohedra. We shall also discuss the additive structure in $H^*(\mathcal{Z}_P)$ in relation with a problem of identifying multiplicative generators of loop homology for $\mathcal{Z}_P$ and Serre's problem on rationality of Poincaré series for local Noetherian rings.

This work is supported by the Russian Science Foundation under grant 14-11-00414.

Pseudo-umbilical real hypersurfaces in complex Grassmannians of rank two

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The complex Grassmannians of rank two (both the compact type and the noncompact type) of complex dimension are Riemannian symmetric spaces equipped with a Kähler structure and a quaternionic Kähler structure.

These geometric structures significantly impose restrictions on the geometry of a real hypersurface in complex Grassmannians of rank two. As an immediate consequence of the Codazzi equation of such submanifolds, the totally umbilicity are too strong to be satisfied by real hypersurfaces in complex Grassmannians of rank two.

We introduce the notion of pseudo-umbilical real hypersurfaces in complex Grassmannians of rank two and give the classification of such real hypersurfaces.

Geometry of loop spaces

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A Riemannian metric on a manifold M induces a family of Riemannian metrics on the loop space LM depending on a Sobolev space parameter s. We compute the connection forms of these metrics and the higher symbols of their curvature forms, which take values in pseudodifferential operators (ΨDOs). These calculations are used to construct Chern-Simons classes on TLM which detect nontrivial elements in the diffeomorphism group of certain Sasakian 5-manifolds associated to Kähler surfaces.

On Lie problem and differential invariats for subgroups of the plane Cremona group

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We develop a new approach to the study of subgroups of Cremona groups. Namely a subgroup of the plane Cremona group may be represented as a symmetry group of differential equations of the form $y'' = F(x, y)$ with polynomial or rational sides, which makes it possible to apply for their study methods of differential invariants theory and geometric theory of differential equations. So we can consider a class of differential equations, which is a "differential model" of the subgroup of Cremona group. Also using algebraic methods in the theory of differential equations we obtain a global classification of such equations.

On quasi contact metric manifolds

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Quasi contact metric manifolds are odd dimensional analogues of quasi Kähler manifolds, they appear on hypersurfaces of quasi Kähler manifolds. In this paper we consider quasi contact metric manifolds and give necessary and sufficient conditions on a quasi contact metric manifold, to be contact metric or K- contact, then we prove that a quasi contact metric manifold is contact, also we give a characterization of contact metric manifolds with respect to quasi contact metric manifolds.

Interior estimates for Poisson type inequality and qc hyperbolic harmonic mappings

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We study quasiconformal (qc) mappings in plane and space and in particular Lipschitz-continuity of mappings which satisfy in addition certain PDE equations (or inequalities). Some of the obtained results can be considered as versions of Kellogg-Warshawski type theorem for qc-mappings. We plan to discuss a major breakthrough concerning the initial Schoen Conjecture (and more generally the Schoen-Li-Wang conjecture) made very recently (Marković and the others including members of Belgrade seminar). Among the other things, as tool we use the interior estimates for Poisson type inequality and try to imply it to study boundary regularity of Dirichlet Eigenfunctions on bounded domains which are C^2 except at a finite number of corners (related to Y. Sinai's question).

Schwarz lemma, the Carathéodory and Kobayashi metrics and applications in complex analysis

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We study Schwarz lemma at the boundary of strongly pseudoconvex domains, and versions of now called the Caratheodory-Cartan-Kaup-Wu theorem, which generalizes the classical Schwarz lemma for holomorphic functions to higher dimensions, and iterations and fixed points of holomorphic mappings.

A new curvaturelike tensor field in an almost contact Riemannian manifold II

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In the last paper, we introduced a new curvaturelike tensor field in an almost contact Riemannian manifold and we showed some geometrical properties of this tensor field in a Kenmotsu and a Sasakian manifold [4].

In this talk, we define another new curvaturelike tensor field, named $(CHR)_2$-curvature tensor in an almost contact Riemannian manifold which is called a contact holomorphic Riemannian curvature tensor of the second type (See (2.2)). Then, using this tensor, we mainly research $(CHR)_2$-curvature tensor in a Kenmotsu and Sasakian manifold. Then we introduce the notion of the flatness of a $(CHR)_2$-curvature tensor and we show that a Kenmotsu and a Sasakian manifold with a flat $(CHR)_2$-curvature tensor is flat. Next, we introduce the notion of $(CHR)_2$-η-Einstein in an almost contact Riemannian manifold. In particular, we show that a Kenmotsu or Sasakian $(CHR)_2$-η-Einstein manifold is η-Einstain. Moreover, we define the notion of $(CHR)_2$-space form and consider this in a Kenmotsu and a Sasakian manifold. Finally, we consider a conformal transformation of an almost contact Riemannian manifold and we get new invariant tensor fields (not the conformal curvature tensor) under this transformation.

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On geodesic mappings

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We study fundamental equations of geodesic mappings of manifolds with affine and projective connection onto (pseudo-) Riemannian manifolds with respect to the smoothness class of these geometric objects. We prove that the natural smoothness class of these problems is preserved. We will speak about geodesic mappings of Einstein spaces.

This is a joint work with Irena Hinterleitner.

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Integrable complex structures on nilpotent Lie algebras

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An almost complex structure J on a Lie algebra $\mathfrak{g}$ ($J : \mathfrak{g} \to \mathfrak{g}$ satisfying $J^2 = -1$) is called integrable (Nijenhuis tensor $N(J)$ vanishes) if

$$N(J) = [JX, JY] - [X, Y] - J[JX, Y] - J[X, JY] = 0, \ \forall X, Y \in \mathfrak{g}.$$

An integrable almost complex structure on the tangent Lie algebra $\mathfrak{g}$ of a real simply connected Lie group G defines a left invariant complex structure on G. If G is nilpotent and $\Gamma \subset G$ is a cocompact lattice, J defines a complex structure on corresponding nilmanifold G/Γ.

We plan to discuss the algebraic constraints on the structure of nilpotent Lie algebra $\mathfrak{g}$ which arise because of the presence of an integrable almost complex structure J on $\mathfrak{g}$.

Salamon studied in [4] 6-dimensional nilpotent Lie algebras admitting integrable complex structure. Goze and Remm have shown [1] that a filiform Lie algebra does not admit any integrable almost complex structure, later Remm and Garcia-Vergnolle extended this result to the class of so-called quasi-filiform Lie algebras [2].

Theorem. *Let* $\mathfrak{g}$ *be a nilpotent Lie algebra endowed with an integrable complex structure and* $\dim \mathfrak{g} \geq 8$*.* $\mathfrak{g}^k = [\mathfrak{g}, \mathfrak{g}^{k-1}]$ *denotes k-th ideal of the descending central sequence of the Lie algebra* $\mathfrak{g}$*. Then we have the following estimates:*

$$\operatorname{codim} \mathfrak{g}^4 \geq 5, \ \operatorname{codim} \mathfrak{g}^6 \geq 8.$$

.

We will provide examples showing that these estimates are sharp.

Remark. *For a filiform Lie algebra* $\mathfrak{g}$ *we have* $\operatorname{codim} \mathfrak{g}^4 = 4$, $\operatorname{codim} \mathfrak{g}^6 = 6$.

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Reillyś type inequality for the Laplacian associated to a density related with shrinkers for MCF

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Let $(\overline{M}, \langle , \rangle, e^{\psi})$ be a Riemannian manifold with a density, and let M be a closed n-dimensional submanifold of $\overline{M}$ with the induced metric and density. We give an upper bound on the first eigenvalue λ_1 of the closed eigenvalue problem for Δ_ψ (the Laplacian on M associated to the density) in terms of the average of the norm of the vector $\vec{H}_\psi + \overline{\nabla}\psi$ with respect to the volume form induced by the density, where $\vec{H}_\psi$ is the mean curvature of M associated to the density e^{ψ}.
When $\overline{M} = \mathbb{R}^{n+k}$ or $\overline{M} = S^{n+k-1}$, the equality between λ_1 and its bound implies that e^{ψ} is a Gaussian density ($\psi(x) = \frac{C}{2}|x|^2$, $C < 0$), and M is a shrinker for the mean curvature flow (MCF) on $\mathbb{R}^{n+k}$.
We prove also that $\lambda_1 = -C$ on the standard shrinker torus of revolution.
Based on this and on the Yauś conjecture on the first eigenvalue of minimal submanifolds of S^n, we conjecture that the equality $\lambda_1 = -C$ is true for all the shrinkers of MCF in R^{n+k}.
This is a joint work with M. Carmen Domingo-Juan.

Angular billiard and algebraic Birkhoff conjecture

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We introduce a new dynamical system which we call Angular billiard. It acts on the exterior points of a convex curve in Euclidean plane. In a neighborhood of the boundary curve this system turns out to be dual to the Birkhoff billiard. Using this system we get new results on algebraic Birkhoff conjecture on integrable billiards.

The results were obtained with Michael Bialy (Tel Aviv).

Polar actions on manifolds of nonpositive curvature

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We give a description of orbits and orbit spaces arising from some polar actions on Riemannian manifolds of nonpositive curvature.

Homotopy invariants of manifolds: Poincare duality and the signature formulae

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The Hirzebruch formula connects the signature of the manifold with some characteristic classes of the manifold. Namely the Hirzebruch formula has the following form:

$$\mathbf{sign}\, X = 2^{2k} \left\langle L(X), [X] \right\rangle,$$

where the class $L(X)$ is so called multiplicative Hirzebruch genus defined by

$$L(X) = \prod_j \frac{t_j/2}{\tanh(t_j/2)},$$

where t_j are formal generators such that $\sigma_k(t_1^2, \ldots, t_n^2) = p_k(X)$ where $p_k(X)$ are the Pontryagin classes.

The crucial property of the Hirzebruch formula is that its left-hand member is expressed exclusively in homotopy terms whereas its right-hand member is an invariant of the smooth structure of the manifold.

This means that if two smooth manifolds are homotopy equivalent and may have different Pontryagin classes nevertheless the Hirzebruch number has the same value.

This circumstance generated many natural problems, some of them resulted deep theorems whereas others are still open.

Inter alia I will speak on Novikov's conjecture (1970) about homotopy invariance of higher signatures and on a short natural proof of the Novikov's theorem (1965) about topological invariance of rational Pontryagin classes which was presented by M.Gromov (1995) on the base of homotopy invariance of some higher signatures. Among last results we will speak how to construct non commutative (symmetric) signature on non compact manifolds with proper action of a discrete group.

This is a joint work with Th. Yu. Popelensky.

Top dense ball packings and coverings in hyperbolic space

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In the classical Euclidean 3−space the so-called Kepler conjecture on the densest packing $\mathbb{E}^3$ with congruent balls (with density $0.74\ldots$) has been recently solved by Thomas Hales by computer, following the strategy of László Fejes Tóth (1953). In the Bolyai-Lobachevsky hyperbolic space we know only a density upper bound (K. Böröczky and A. Florian (1964)), realized (only) by horoballs in ideal regular simplex arrangement with density $0.85\ldots$, and the realization is not unique (R.T. Kozma and J. Szirmai [2]). With proper balls we are far from this packing upper bound, and there is no real chance yet for the more difficult ball covering problem in $\mathbb{H}^3$.

Our aim in this work is a systematic computer experiment to attacking both problems for packing and covering by a construction scheme. These ball arrangements will be based on complete (or extended) Coxeter orthoscheme groups, generated by plane reflections.

E.g. the Coxeter-Schläfli symbol $(u, v, w) = (5, 3, 5)$ describes first the characteristic orthoscheme of a regular dodecahedron (as $(5, 3, \,.\,)$ refers to it) with dihedral face angle $2\pi/5$ (as $(\,.\,, \,.\,, 5)$ indicates it). This dodecahedron - by its congruent copies - fills $\mathbb{H}^3$ just by the reflections in the side faces of the above orthoscheme (characterized also by a Coxeter-Schläfli matrix, scalar product of signature $(+++-)$, etc.). This orthoscheme $A_0A_1A_2A_3$ has also a half-turn symmetry $0 \leftrightarrow 3$, $1 \leftrightarrow 2$ that extends to the complete symmetry group of the $\mathbb{H}^3$ tiling. Not surprisingly, we get the H3 tiling with the hyperbolic football (the Archimedean solid $\{5, 6, 6\}$) as in the earlier works [3, 4] of the first author. The central ball (centred in A_3 or in A_0) in the above football solid has **the packing density** $0.771\ldots$, as a maximal density so far, just discovered now. This hyperbolic football provides also **the covering density** $1.369\ldots$, minimal so far, as brand new observation.

For this and the analogous generalized further series, a volume formula of orthoscheme by N.I. Lobachevsky (1837) was needed, that has been extended to complete (or truncated) orthoschemes by R. Kellerhals [1]. The second author intensively worked on its computer program (see e.g. [5]). Thus we get a large list of good (top!?) constructions as for packing densities as for covering ones as well, together with their metric data. For these the ball centre also varies on the surface of the (truncated) orthoscheme, together with the ball radius. So we have to implement large computations, indeed.

This is a joint work with Jenő Szirmai.

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Lagrangian submanifolds in the nearly Kähler $S^3 \times S^3$ from minimal surfaces in S^3

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Nearly Kähler (abbrev. NK) manifolds are almost Hermitian manifolds with almost complex structure J satisfying that $\tilde{\nabla} J$ is skew-symmetric. Butruille [1] proved that the only homogeneous 6-dimensional NK manifolds are the NK $\mathbb{S}^6$, $\mathbb{S}^3 \times \mathbb{S}^3$, the complex projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1)\times U(1)$. A natural and interesting question for the above four NK manifolds is to investigate their almost complex submanifolds and their Lagrangian submanifolds. As NK manifolds are an important class of Hermitian manifolds, we can consider Lagrangian submanifolds more generally in almost Hermitian manifolds. We say that such a submanifold is Lagrangian if the almost complex structure J interchanges the tangent and the normal spaces and if the dimension of the submanifold is half the dimension of the ambient manifold.

In the present talk, we refer to the study of minimal Lagrangian submanifolds M in the NK $\mathbb{S}^3 \times \mathbb{S}^3$ described by $g \mapsto f(g) = (p(g), q(g))$. It is known that in order to describe such submanifolds the so called angle functions (see [2]) play an important role.

Here we describe the Lagrangian submanifolds of the NK $\mathbb{S}^3 \times \mathbb{S}^3$ whose angle functions are $\theta_1 = \frac{\pi}{3}$, $\theta_2 = \alpha + \frac{\pi}{3}$ and $\theta_3 = -\alpha + \frac{\pi}{3}$. Geometrically this corresponds to the Lagrangian immersions for which the map p is not an immersion.

We show that M can be identified with an open part of a frame bundle over a minimal surface N. Moreover the immersion is determined by an additional differential equation. Moreover, we study as well the reverse problem, in the cases when the minimal surface is totally geodesic or not.

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Magnetic maps

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This talk is based manly on some work with Jun-ichi Inoguchi.

We introduce the notion of magnetic maps between Riemannian manifolds. They are generalizations of both magnetic curves and harmonic maps. We provide some fundamental examples of magnetic maps. Furthermore, we study some classes of magnetic surfaces in Euclidean 3-space. Then we produce examples of magnetic maps, having as either source or target manifold the tangent bundle of a Riemannian manifold equipped with several Riemannian metrics. In particular we study when the canonical projection, a vector field and the tangent map are, respectively, magnetic maps.

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Bochner curvature tensor of locally conformal Kähler space forms

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The notion of a locally conformal Kähler manifold (an l.c.K-manifold) in a Hermitian Geometry has been introduced by I. Vaisman in 1976. In this work, the Bochner curvature tensor in l.c.K-manifolds and l.c.K-space forms are presented. Moreover, some properties of the Bochner curvature tensor in an l.c.K-space form are studied.

This is a joint work with Zerrin Şentürk.

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Infinitesimal deformations of subspaces of generalized Riemannian spaces

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This is joint work with Ljubica Velimirović and Nenad Vesić.

We study infinitesimal deformations of subspaces of generalized Riemannian spaces and examine variations of some important geometrical magnitudes. Specially, we consider geodesic infinitesimal deformations and obtain necessary and sufficient conditions which provide the existence of that kind of deformations of generalized Riemannian spaces.

Siacci's and the areal basis resolution of the acceleration vector for a space curve in Minkowski 3-space

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We give a kinematic decomposition, known as the Siacci's resolution, of the acceleration vector along a space curve in Minkowski 3-space, depending on the causal character of the curve. We also obtain the corresponding decomposition of the acceleration vector in terms of the areal basis of the curve and give some examples.

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Topology of non-compact integrable Hamiltonian systems: first steps

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The theory of topological invariants for Liouville integrable Hamiltonian systems created by A. T. Fomenko and his school [1-3] allows to investigate the topology of the corresponding Liouville foliations and the spaces of trajectories of such systems in the compact case (i.e. when all the leaves of the Liouville foliation are compact). However there exist many natural examples of integrable systems with non-compact Liouville foliations. In the talk we discuss first attempts to generalize the Fomenko theory for non-compact integrable systems. First of all, we focus on the most simple case: Hamiltonian systems with one degree of freedom. Every such system is Liouville integrable, since its Hamiltonian function suffices for integrability. Under some conditions these systems can be easily classified up to topological equivalence in the terms similar to those in the compact case. At the same time, in the problem of classification of such systems up to conjugacy some new effects appear.

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H-contact unit tangent sphere bundles

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A contact metric manifold is said to be H-contact, if the characteristic vector field is harmonic. We prove that the unit tangent bundle of a Riemannian manifold M equipped with the standard contact metric structure is H-contact if and only if M is 2-stein.

This is a joint work with J. H. Park.

The moduli space of psuedo holomorphic disks with jumping Lagrangian boundary conditions

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We discuss the moduli spaces of perturbed pseudo–holomorphic disks with boundary on cleanly intersecting Lagrangian submanifolds. We show that these moduli spaces have the structure of manifolds with corners and we compute the dimension in special cases. We also explain how to glue two pseudo–holomorphic disks.

Geometrical coding of digital images

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Automatic analysis, complexity measuring and similarity determining of digital images is important part of modern computer geometry. One of the main problems here is a problem of effective coding of digital image which allows fast and robust analysis and compare algorithms. The new approach of geometrical coding by special 2 dimensional surfaces is suggested. With this approach digital images recognition problems could be formulated in terms of geometrical characteristics of coding surfaces.

Cauchy-Kowalevski's theorem applied for counting geometric structures

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The study is inspired by the recent paper of Z. Dusek and O.Kowalski [1]. Roughly speaking the question is how many structures of a prescribed type there exist. By a satisfactory answer we mean a theorem saying that the set of such structures is parametrized by some family (finite) of arbitralily chosen functions. A local version of the question is considered. It turns out that the theorem of Cauchy-Kowalevski can be used in answering it. Of course, using this tool implies that we must restrict to analytic structures. But the advantage is that the tool belongs to the fundamentals of mathematics and the procedure of getting structures is explicit modulo solving a Cauchy-Kowalevski system of differential equations. On the other hand, it seems that the method fits only very special situations.

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[2] Mikulski W, Opozda B. Cauchy-Kowalevskiś theorem applied for counting geometric structures. arXiv: 1605.06248

Surfaces in $\mathbb{R}^7$ obtained from harmonic maps in S^6

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We will discuss the local geometry of surfaces in the 7- dimensional Euclidean space obtained from harmonic maps from a Riemannian surface Σ into S^6. In this setting, the harmonicity of a smooth map $\varphi : \Sigma \to S^6$ amounts to the closeness of the differential 1-form $\omega = \varphi \times *d\varphi$, where $\times$ stands for the 7-dimensional cross product. This means that we can integrate on simply- connected domains in order to obtain a map $F : \Sigma \to \mathbb{R}^7$. By applying methods based on the use of harmonic sequences, we will characterize the conformal harmonic immersions φ whose associated immersions F in $\mathbb{R}^7$ belong to certain remarkable classes of surfaces, namely: minimal surfaces; pseudo-umbilical surfaces; surfaces with parallel mean curvature vector field; isotropic surfaces.

Polyhedral products and commutator subgroups of right-angled Artin and Coxeter groups

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We construct and study polyhedral product models for classifying spaces of right-angled Artin and Coxeter groups, general graph product groups and their commutator subgroups. By way of application, we give a criterion of freeness for the commutator subgroup of a graph product group, and provide an explicit minimal set of generators for the commutator subgroup of a right-angled Coxeter group.

This is a joint work with Yakov Veryovkin.

On Deszcz symmetries of generalised Wintgen ideal Lagrangian submanifolds

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In this talk different kinds of pseudosymmetry curvature conditions in the sense of Deszcz are considered for Lagrangian submanifolds in complex space forms. For Lagrangian submanifolds M^n in complex space forms $\tilde{M}^m(4c)$, an inequality relating the main intrinsic and extrinsic scalar invariants, namely the normalised scalar curvature (intrinsic invariant) and the squared mean curvature and the normalised scalar normal curvature of M in the ambient space $\tilde{M}$ (extrinsic invariants) is called *generalised Wintgen inequality.* And a Lagrangian submanifold M^n is said to be *generalised Wintgen ideal Lagrangian submanifold* of $\tilde{M}^m(4c)$ when it realises at everyone of its points the equality in such inequality.

Characterizations based on some basic Deszcz symmetries of such generalised Wintgen ideal Lagrangian submanifols are given.

Completions of upper-triangular matrices to Kato nonsingularity

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In this paper we consider the problem of completion of the uppertriangular operator matrix $\begin{bmatrix} A & ? \\ 0 & B \end{bmatrix}$, where $A \in \mathcal{B}(\mathcal{H})$ and $B \in \mathcal{B}(\mathcal{K})$, to Kato nonsingular operators and completely solve it in each of the following cases: one of the operators A or B is Kato nonsingular; B is injective; A is with dense range; B is with finite ascent; A is with finite descent; $0 \notin int(\sigma_p(B))$; $0 \notin int(\sigma_{cp}(A))$. In particular, the results generalize and complete some of the previously obtained concerning the same problem.

This is a joint work with D. S. Cvetković-Ilić.

On holomorphically projective mappings of parabolic Kähler manifolds

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We study with H. Chudá, J. Mikeš, and M. Shiha fundamental equations of holomorphically projective mappings of parabolic Kähler spaces (which are generalized classical, pseudo- and hyperbolic Kähler spaces) with respect to the smoothness class of metrics, see [1, 2]. We show that *holomorphically projective mappings preserve the smoothness class of metrics.*

We remind, that an n-dimensional (pseudo-) Riemannian manifold (M, g) is called an *m-parabolic Kähler manifold*, if beside the metric tensor g, a tensor field F of a rank $m > 1$ of type $(1,1)$ is given on the manifold M_n, such that the following conditions hold: $F^2 = 0$, $g(X, FX) = 0$, $\nabla F = 0$, where X is an arbitrary tangent vector, ∇ denotes the covariant derivative.

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On almost geodesic mappings of the second type between manifolds with non-symmetric linear connection

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We derive two mixed systems of Cauchy type in covariant derivatives of the first and second kind for almost geodesic mappings of the second type between manifolds with non-symmetric linear connection. Also, we consider a particular class of these mappings determined by the condition $\nabla F = 0$, where ∇ is the symmetric part of non-symmetric linear connection ${}^1\nabla$ and F is the affinor structure. Finally, we examine some invariant geometric objects with respect to these mapping.

Differential-based geometry modeling and processing of meshes

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With the unifying eye of integrability we will look at a diverse set of recent geometry processing algorithms. Several current geometry algorithms can be formulated as systems of first order PDEs on surface or volume meshes in a discrete differential geometric framework. This includes problems such as image and geometry retargeting, Laplacian modeling, surface and volume parametrization, minimal surface computation and many others.

The underlying mathematical techniques rely on a consistent discrete differential geometry framework including concepts such as discrete differential forms and discrete Hodge decomposition. Using several example problems we show that these integrability issues are fundamental for a wide range of local and global problems in geometry processing. Applications to various problems in scientific computing, computer aided design, architecture and computer graphics are shown.

Bi-Hermitian metrics on Kato surfaces

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The problem of classification of bi-Hemitian structures on four-manifolds was introduced by S. Salamon in the 90's: given a fixed compact Hermitian surface (S, g, J) - where g denotes a Riemannian metric, is it possible to find a different complex structure I which is still Hermitian with respect to the same metric g? The first instance of this phenomenon are hypercomplex structures, for example hyperKähler surfaces in which case however one actually gets an infinite number of such I's. The problem of finding and classifying bi-Hermitian surfaces is clearly a conformally invariant one and has proved to be connected with at least two interesting fields of active research in complex geometry. The first one is more classical - initiated by I. Vaisman and F. Tricerri

- and is the problem of finding locally conformally Kähler structures, usually abreviated with l.c.K. The second one is more recent - due to N. Hitchin and M. Gualtieri - who introduced generalized Kähler geometry in the early 2000's and showed that such a structure always gives rise to a bi-Hermitian structure.

Compact Kähler surfaces are naturally divided in two classes: surfaces of Kähler type and non-Kähler type. As the Kähler case has recently been classified by the work of Hitchin, Gualtieri and Goto, our main contributions are concerned with the non-Kähler case.

Our main contributions are the following:

1. A complete classification of surfaces admitting a bi-Hermitian structure such that the anti-canonical divisor is disconnected. This was achieved via a twistor construction also produced as a by-product new antiself-dual metrics as well as new l.c.K. metrics, stimulating novel interest and ideas in the subject eventually leading to a power result of M. Brunella who solved the l.c.K. existence question for all known compact complex surfaces.

2. A complete classification of bi-Hermitian surfaces with connected anti-canonical divisor. This new result is achieved thanks to the interplay - discovered by V. Apostolov-G. Dloussky - between l.c.K. metrics and bi-Hermitian structures.

The existence problem is now completely solved for all surfaces except for a class of Kato surfaces which are called *intermediate* for which our existence result is complete only up to a logarithmic deformation. We complete the classification of compact complex surfaces with two different complex structures orthogonal to a fixed Riemannian metric by considering the case of Kato surfaces admitting a connected numerical anti-canonical divisor.

Joint work with Prof. A. Fujiki.

Toda conjecture mod $p > 3$

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Toda constructed and used (see "On exact sequences in Steenrod algebra mod 2", Memoirs of the College of Science, University of Kyoto. Series A: Mathematics 31 (1958), no. 1, 33-64) some exact sequences for quotients of the Steenrod algebra $\mathcal{A}_2$. He also conjectured the exactness of the sequence

$$\mathcal{A}_2 \xrightarrow{\varphi_r} \mathcal{A}_2/\mathcal{A}_2\mathcal{S}_{r-2} \xrightarrow{\varphi_r} \mathcal{A}_2/\mathcal{A}_2\mathcal{S}_{r-1} \tag{1}$$

where $\varphi_r(x) = x \cdot Sq^{2^r}$ and $\mathcal{S}_k$ is the subalgebra generated by the elements Sq^1, Sq^2, $Sq^4, \dots, Sq^{2^k}$. The conjecture was proved by Wall in "Generators and relations for the Steenrod algebra", Ann. of Math. (2) 72 (1960), 429Ű444.

Consider prime $p > 3$. Let a and b be integers such that $a, b \in [1, p-1]$ and $a + b = p$. Define $\alpha_i : \mathcal{A}_p \xrightarrow{\mathcal{A}_p} /\mathcal{A}_p\mathcal{S}_{r-2}$ by $\alpha_1(x) = x \cdot P^{ap^r}$ and $\alpha_2(x) = x \cdot (P^{p^r})^a$ and also $\beta_i : \mathcal{A}_p/\mathcal{A}_p\mathcal{S}_{r-2} \xrightarrow{\mathcal{A}_p} /\mathcal{A}_p\mathcal{S}_{r-1}$ by $\beta_1(x) = x \cdot P^{bp^r}$ and $\beta_2(x) = x \cdot (P^{p^r})^b$.

One can show that for any $a + b = p$ and $i, j \in \{1, 2\}$ there are well defined homomorphisms of quotients

$$\mathcal{A}_p \xrightarrow{\alpha_i} \mathcal{A}_p/\mathcal{A}_p\mathcal{S}_{r-2} \to \beta_j\mathcal{A}_p/\mathcal{A}_p\mathcal{S}_{r-1}. \tag{2}$$

Moreover one has $\beta_j \circ \alpha_i = 0$.

Exactness of the sequence (2) is the natural generalization of the Toda conjecture for the case $p > 2$.

Theorem. (a) The element $2Z^r_{r-1}Z^{r-1}_{r-2} - Z^r_{r-2}Z^{r-1}_{r-1}$ belongs to the kernel β_j, but does not belong to the image of α_i.

(b) For $\beta(x) = x \cdot P^{(p-1)p^r}$ and $\alpha(x) = x \cdot P^{p^r}$ the sequence is exact in lower grading.

(c) In grading $2(p-1)k = 2(p-1)(2p^{r-1} + p^{r-2})$ the quotient $ker\ \beta/im\ \alpha$ is of dimension one.

The proof is based on the results from the paper "On monomial bases in the mod(p) Steenrod algebra" by D. Yu. Emelyanov, Th. Yu. Popelensky, Journal of Fixed Point Theory and its Applications. 2015, vol. 17, no. 2, 341-353

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Lightlike hypersurfaces of golden semi-Riemannian manifolds

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In this paper, we study lightlike hypersurfaces of Golden semi-Riemannian manifolds.

An m-dimensional semi-Riemannian manifold $(\tilde{M}, \tilde{g}, \tilde{P})$ is called a golden semi-Riemannian manifold if the (1,1)-tensor field $\tilde{P}$ on $\tilde{M}$ is a golden structure (i.e. $\tilde{P}^2 = \tilde{P} + I$) and $\tilde{g}(\tilde{P}X, Y) = \tilde{g}(X, \tilde{P}Y)$ for every tangent vector fields $X, Y \in \Gamma(TM)$. We investigate several properties of lightlike hypersurfaces of Golden Semi- Riemannian manifolds. We define invariant lightlike hypersurfaces, screen semi- invariant lightlike hypersurfaces and radical anti-invariant lightlike hypersurfaces. We show that there is no radical anti-invariant lightlike hypersurface of Golden semi-Riemannian manifold. In particular, we obtain some results for screen semi-invariant lightlike hypersurfaces of Golden Semi-Riemannian Manifolds. We also give some special examples.

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Families of plaited polyhedra as knots and links

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We analyzed some families of plaited polyhedra from the knot-theory point of view. These families of plaited polyhedra, discovered by P. Gerdes, occurring as African dance rattle capsules. Every plaited polyhedron can be derived from a shadow of some knot and link, by using dual of the shadow. We introduced different construction methods based on families of knots and links or their corresponding braid families, resulting in families of plaited polyhedra.

A Nahm transform between G2-monopoles and 5-dimensional Instantons

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The moduli space of anti self-dual (ASD) instantons of a oriented 4-manifold is an interesting space which carries information about the smooth structure of the manifold, and is, e.g., used by in his groundbreaking work on the geometry of 4-manifolds.

Reductions of the ASD-equations to lower dimensions gives defines the Bogomonly monopoles, Hitchin equations, Nahm equations and ADHM-data, which are all subject of ongoing studies. Nahm and Hitchin defined a **Nahm transform** that relates the moduli space of Nahm equations with the moduli space of Bogomolny monopoles.

In their article *Guage theory in Higher Dimensions II*, Donaldson and Segal argue that there are similar theories in higher dimensions, strongly related to Berger's holonomy classification. In particular there is a 7-dimensional theory called G_2-monopoles, and a 5-dimensional instanton theory (Haydys-Witten equations). I work on a relation between those theories that is similar to the Nahm transform.

In my talk I'd like to give a short overview of the theory mentioned above and give a description of the relation mentioned.

The Einstein-Hilbert type action on foliated metric-affine manifolds

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We discuss recent results (joint with Tomasz Zawadzki): variation formulas for extrinsic geometric quantities for variations of pseudo-Riemannian metric and connection on almost-product (e.g. foliated) metric-affine manifolds, and applications to study the scalar curvature type actions (e.g. the total mixed and extrinsic scalar curvatures) – analogs of the classical Einstein-Hilbert action.

Categorical description of plant morphogenesis

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Incredible complexity, variability and beauty of living plants and their organs are caused by partially deterministic successive activity of the formative tissues called meristems. There is finite number of developmental processes and types of traffic relations at the cellular level common for all plants, which gives rise to the species-specific geometric structure of their organs and tissues. In this work the formalistic description of plant morphogenesis is presented. Spatial structure of tissues was described combinatorially as a simplicial object in the category of sets and using Haken approach by S. Matveev as a three dimensional manifold. Dynamical properties of the structure such as developmental changes of tissues and traffic relations between cells were described using the Petri nets formalism. The categorical language of data representation and analysis enabled to reveal basic principles of the programming of plant form and shape development.

CMC hypersurfaces with a canonical principal direction in space forms

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A hypersurface $M \subset \overline{M}$ of the space form $\overline{M}$ has a canonical principal direction (CPD) relative to the closed and conformal vector field Z of $\overline{M}$ if the projection $Z^{\top}$ of Z to M is a principal direction of M. We show that CPD hypersurfaces with constant mean curvature are foliated by isoparametric hypersurfaces.

On generalised Wintgen ideal Legendrian submanifolds

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For all submanifolds M^n of all real space forms $\tilde{M}^{n+m}(c)$ with constant sectional curvature c, for all dimensions $n \geq 2$ and for all co–dimensions $m \geq 1$, the so–called *Wintgen inequality* was conjectured by De Smet, Dillen, Verstraelen and Vrancken (1999). The conjecture, also known as the DDVV conjecture, was proven by Ge and Tang (2008) and by Lu (2011), independently. I. Mihai obtained the Wintgen inequality, also known as the *generalised Wintgen inequality*, for Lagrangian submanifolds in complex space forms (2014) as well as for Legendrian submanifolds in Sasakian space forms (2015), and also characterised the corresponding equality cases. Submanifolds M which satisfy the equality in these optimal general inequalities are called *generalised Wintgen ideal submanifolds* in the ambient space $\tilde{M}$. For *generalised Wintgen ideal Legendrian submanifolds* M^n of Sasakian space forms $\tilde{M}^{2m+1}(c)$, in the present article, we will show some properties concerning their *Deszcz symmetry* and their *Roter type*. We also show that for such generalised Wintgen ideal Legendrian submanifolds, the (intrinsic) *Ricci principal directions* and the (extrinsic) *Casorati principal directions* coincide.

Computing Aluthge and Duggal transform of polynomial matrices with complex coefficients

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We develop algorithms for the symbolic computation of the Aluthge and Duggal transform of a polynomial matrix. Thus, PSVD by PQRD algorithm from [Foster et al *An algorithm for calculating the QR and singular value decompositions of polynomial matrices.* IEEE Transactions on Signal Processing, 58(3) : 1263-1274, 2009] is used in order to avoid square-root elements in transformed matrices, which is essential for symbolic computations. Numerical examples are investigated as well to illustrate the benefits of using the introduced algorithms.

Generalized graphical method of linear programming

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This talk is based on joint research with Ljubica S. Velimirović.

Linear programming (LP) is used for solving all problems that can be represented by a system of linear equations. Graphical method is often limited to LP problems involving two or three decision variables and a limited number of constraints due to the difficulty of the graphical representation, which restricts the use to real-world problems. We develop the extension of the classical graphical method on n decision variables. Therefore, analytical method for n variable case graphical method is introduced, and we prove that obtained solutions of LP problem are optimal. Some details about the implementation of graphical method for solving LP problems in n variables are discussed, and numerical examples are provided to illustrate the introduced algorithm.

Solving indefinite least-squares problem using generalized inverses and Recurrent Neural Network

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The matrix J in a pseudo-Euclidean space is defined by

$$J = \pm \begin{bmatrix} I_k & 0 \\ 0 & -I_{n-k} \end{bmatrix} = I_{k,n-k} \in \mathbb{C}^{n\times n},\ k \leq n, \tag{1}$$

where I_n denotes the $n \times n$ identity matrix. Then J is the metric tensor satisfying $Ju = \pm(u_1, \ldots, u_k, -u_{k+1}, \ldots, -u_n)$, where $u = (u_1, u_2, \ldots, u_n) \in \mathbb{C}^n$. In the particular case $k = 1$ the metric matrix (1) defines Minkowski inner product. Our goal is to find solutions of the solution of indefinite least-squares problems of the general form

$$\min_x\ (Ax - b)^T J(Ax - b).$$

Firstly, a correlation between the indefinite least-squares problems and particular classes of generalized inverses are investigated. Using various representations of these generalized inverses, two gradient-based recurrent neural networks (RNNs) for their computation are defined.

Nonlocal modification of the Einstein theory of gravity

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In this contribution we consider model of nonlocal gravity without matter, given by the following action

$$S = \int \Big(\frac{R-2\Lambda}{16\pi G} + C(R+R_0)^m \mathcal{F}(\Box)(R+R_0)^m\Big)\sqrt{-g}d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alembertian $\Box$, Λ is cosmological constant and C,R_0, m are real constants. The corresponding Einstein equations of motion are derived and presented. Using ansatze we can solve equations of motion and get some cosmological solutions.

This is a joint work with I. Dimitrijević, B. Dragovich, A. S. Koshelev and Z. Rakić.

Liouville type theorems for Riemannian twisted and warped products

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The *doubly twisted product* ${}_{\lambda_1}M_1 \times_{\lambda_2} M_2$ is the product manifold $M_1 \times M_2$ furnished with the Riemannian metric $g = \lambda_1^2\pi_1^* g_1 + \lambda_2^2 \pi_2^* g_2$ where $\lambda_i : M_1 \times M_2 \to \mathbb{R}$ is a positive function and $\pi_i : M_1 \times M_2 \to M_i$ is the canonical projection for an arbitrary $i = 1, 2$ (see [1]). Then we can formulate a Liouville type theorem for doubly twisted products.

Theorem 1. *Let (M, g) be a doubly twisted product ${}_{\lambda_1}M_1 \times_{\lambda_2} M_2$ of some Riemannian manifolds (M_1, g_1) and (M_2, g_2). If (M, g) is a complete, noncompact and oriented Riemannian manifold (M, g) with $\|\pi_{2*}(\nabla log\lambda_1) + \pi_{1*}(\nabla log\lambda_2)\| \in L^1(M, g)$ and nonpositive mixed scalar* *curvature* s_{mix}*, then* λ_1 *and* λ_2 *are constants* C_1 *and* C_2*, respectively, and* (M, g) *is the direct product* $M_1 \times M_2$ *of* $(M_1, \overline{g}_1)$ *and* $(M_2, \overline{g}_2)$ *for* $\overline{g}_1 = C_1^2 g_1$ *and* $\overline{g}_2 = C_2^2 g_2$*.*

Let $f : (M, g) \to (\overline{M}, \overline{g})$ be a *projective submersion* with geodesically complete fibres, then (M, g) is isometric to a twisted product $M_1 \times_\lambda M_2$ (see [2]). In this case, the following corollary is true.

Corollary. *Let* (M, g) *be an n-dimensional complete, noncompact and simply connected Riemannian manifold and* $f : (M, g) \to (\overline{M}, \overline{g})$ *be a projective submersion onto another m-dimensional* $(m < n)$ *Riemannian manifold* $(\overline{M}, \overline{g})$ *with geodesically complete fibres. If the mixed sectional curvature of* (M, g) *is nonnegative then it is isometric to a direct product* $M_1 \times M_2$*.*

The *doubly warped product* ${}_{\lambda_1}M_1 \times_{\lambda_2} M_2$ is the twisted product ${}_{\lambda_1}M_1 \times_{\lambda_2} M_2$ for the case when $\lambda_1 : M_2 \to \mathbb{R}$ and $\lambda_2 : M_1 \to \mathbb{R}$ (see [1]). As a corollary of Theorem 1 we formulate the following Liouville-type theorem.

Theorem 2. *Let* (M, g) *be a doubly warped product* ${}_{\lambda_1}M_1 \times_{\lambda_2} M_2$ *of complete Riemannian manifolds* (M_1, g_1) *and* (M_2, g_2) *such that* $inf\lambda_1 > 0$ *or* $inf\lambda_2 > 0$ *and the mixed scalar curvature* s_{mix} *is nonpositive. If the gradient of* $log(\lambda_1\lambda_2)$ *has integrable norm, then* λ_1 *and* λ_2 *are constants* C_1 *and* C_2*, respectively, and* (M, g) *is the direct product of* $(M_1, \overline{g}_1)$ *and* $(M_2, \overline{g}_2)$ *for* $\overline{g}_1 = C_1^2 g_1$ *and* $\overline{g}_2 = C_2^2 g_2$*.*

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Spectral Cartan analysis of anisotropic structures based on Garner-type dynamics

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The higher order tensor spectral analysis was extensively developed in the last two decades, whether most of the results imply tensors considered on the Euclidean space. We point out the specific differences which occur, while considering the more general anisotropic metric framework, and propose several potential extensions. We apply the extended spectral analysis to determine the characteristics of the Cartan tensor in Randers and 4-th root Finsler structures, which are designed to trace out the average evolution in Garner cancer population dynamical systems.

Upgrading the axiomatic system to n-dimensional space

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There are three ways to approach the Euclidean geometry of four or more dimensions: axiomatic, algebraic (or analytical), and intuitive. Of course, only the first two can be formalized. While the algebraic method is well developed, that is not the case with the axiomatic. The terms as "hyperplane", "hyperspace", "n-flat" were used even in 1929 by Sommerville. Also, he described incidence relations between them. But, there is still no precise axiomatic system describing n-dimensional space.

Isoperimetric inequality and related problems

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Let Γ be a simple closed curve in the Euclidean plane and Ω is the interior of Γ. If L is the length of Γ and A is the area of Ω, then the isoperimetric inequality states that

$$4\pi A \leq L^2. \tag{1}$$

Equality holds in (1) if and only if Γ is a circle.

There are many proofs and many generalizations of inequality (1).

Here, we discuss the isoperimetric-type inequalities for subharmonic functions on the polydisk, capacity, the transportation approach and related problems. In particular, we consider new approaches to the exact estimate of the isoperimetric coefficient in the plane and the space (for a review of the subject, see [1]).

[1] Mateljević M. Isoperimetric-type inequalities for subharmonic functions on the polydisk, capacity, transportation approach, and related problems. Filomat. 2015; 29(2): 275-302.

Classification of the left-invariant metrics on $\mathbb{R}H^n$

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It is a well known fact that the real hyperbolic space $\mathbb{R}H^n$ with the standard Riemannian metric of constant negative curvature has a structure of a Lie group such that the metric is left-invariant. We give the full classification of left-invariant metrics of an arbitrary signature on the Lie group corresponding to the real hyperbolic space. We show that all metrics have constant sectional curvature and that they are geodesically complete only in the Riemannian case.

This is a joint work with Srđan Vukmirović.

The Moutard transformation of two-dimensional Dirac operators and the Mobius geometry

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We show that the Moutard type transformation of two-dimensional Dirac operators has a geometrical meaning: it maps the potential U of a Weierstrass representation of a surface S into the potential of its inversion, and apply that to constructing blow-up solutions of the modified Novikov–Veselov equation.

Computations of Weighted Moore-Penrose inverse of rational matrices using Database storage system

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A unified approach for computations of Weighted Moore-Penrose inverse of rational matrices using database storage system are developed. We propose a client/server-based model for computing the weighted Moore-Penrose inverse using the partitioning method along with the implementation and experimental support. Our main motivation was to provide better access compared to our previous work, as well as other studies, primarily in the field of symbolic matrix calculations. Specifically, we provide a new approach for computation generalized weighting MP inverse matrix based on object-oriented programming (OOP) with the technique of storing with NoSQL Database Approach.

Geometric formality of manifolds in small dimensions

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A closed oriented manifold M is said to be geometrically formal if it admits a Riemannian metric for which all wedge products of harmonic forms are again harmonic forms. Geometrically formal manifolds are formal in the sense of rational homotopy theory. If M is a geometrically formal manifold of dimension ≤ 4, then it is known that M has the real cohomology algebra of a compact symmetric space. We consider the question of geometric formality for simply connected manifolds of dimension $5, 6$ and 7. We also assume these manifolds to be rationally elliptic, which simplifies their real cohomology structure. The manifolds of dimensions 5 and 6 are known to be rationally formal and we prove that any such geometrically formal manifold has the real cohomology structure of a symmetric space. In dimension 7 we study this question on homogeneous spaces and biquotients.

Some classification results on biconservative hypersurface in pseudo-Euclidean spaces

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A hypersurface M in a semi-Euclidean space is called biconservative if the tangential component of $\Delta^2 x = 0$, where x is the position vector of M and Δ is the Laplace operator. This condition is equivalent to being principle direction of gradient of the mean curvature of M with corresponding principal curvature a constant multiple of the mean curvature. In this talk, we would like to present some of recent classification results on biconservative hypersurfaces that we have obtained.

On the existence of a new connection on a Weyl manifold

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On a Weyl manifold, the existence of a semi-symmetric recurrent metric connection is proved. We study curvature invariants and their characteristics of manifolds having this connection. Metric tensor of constant scalar curvature and zero curvature spaces in terms of metric reccurency and torsion components are presented, especially. The geodesics structure of the spaces are also studied.

This is joint work with F. Özdemir.

[1] Eisenhart L. P. Non-Riemannian Geometry. New York: The American Mathematical Society Publishing. 1927.

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Geometry of Gromov-Hausdorff space and optimal networks

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In this talk we deal with the hyperspace of metric compact spaces (considered up to isometry) endowed with Gromov-Hausdorff metric. We give a review on known geometrical properties of this hyperspace, and also observe some recent results of the author and his colleagues devoted to this subject. In particular, we discuss a relation between Steiner minimal trees and one-dimensional Gromov minimal fillings constructed for boundaries in the hyperspace.

Qualitative behavior of integrable Hamiltonian systems in a neighbourhood of the saddle-saddle point

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We discuss how to define the foliation in the neighbourhood of the 4-dimensional singular point in terms of Fomenko-Zishang invariants of the neighbourhood boundary.

Note on infinitesimal bending of knots

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This is a joint work with Louis H. Kauffman, University Illinois Chicago, USA, Marija S. Najdanović, Faculty of Science and Mathematics, University of Niš, Serbia, and Svetozar Rančić, Faculty of Science and Mathematics, University of Niš, Serbia.

This talk is devoted to a study of the energy of knots and the infinitesimal bending of knotted curves. Variations of the related geometric magnitudes will be studied. Willmore energy, the total curvature, the total torsion, as well as the total normalcy are obtained. Some examples are visualized.

Pontryagin algebras of some moment-angle complexes

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We consider the problem of describing the Pontryagin algebra (loop homology) of moment-angle complexes and manifolds. The moment-angle complex $\mathcal{Z}_\mathcal{K}$ is a cell complex built of products of polydiscs and tori parametrised by simplices in a finite simplicial complex $\mathcal{K}$. It has a natural torus action and plays an important role in toric topology. In the case when $\mathcal{K}$ is a triangulation of a sphere, $\mathcal{Z}_\mathcal{K}$ is a topological manifold, which has interesting geometric structures.

Generators of the Pontryagin algebra $H_*(\Omega\mathcal{Z}_\mathcal{K})$ when $\mathcal{K}$ is a flag complex were described in the work of Grbic, Panov, Theriault and Wu. Describing relations is often a difficult problem, even when $\mathcal{K}$ has a few vertices. Here we describe these relations in the case when $\mathcal{K}$ is the boundary of pentagon or hexagon. In this case, it is known that $\mathcal{Z}_\mathcal{K}$ is a connected sum of products of spheres with two spheres in each product. Therefore $H_*(\Omega\mathcal{Z}_\mathcal{K})$ is a one-relator algebra and we describe this one relation explicitly, therefore giving a new homotopy-theoretical proof of McGavranś result. An interesting feature of our relation is that it includes iterated Whitehead products, which vanish under the Hurewicz homomorphism. Therefore, the form of this relation cannot be deduced solely from the result of McGavran.

Generalized Bochner Tensor

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Riemannian space $\mathbb{GR}_N$ are studied in this paper. There are two main results obtained inhere. The first result are conclusions about conformal mappings defined on semi-symmetric and quarter-symmetric spaces. The second result is a generalization of a Bochner tensor.

Some Relations Between Generalizations of Weyl Projective Tensor

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A method for connecting generalizations of Weyl projective tensor in a space $\mathbb{GA}_N$ and this generalization in an associated space $\mathbb{A}_N$ is presented in this paper. It is simplified a computation process in the case of generalization of a projective tensor in this way. This method is applied on third type almost geodesic mappings of the space $\mathbb{GA}_N$.

On complexity and Turaev-Viro invariants of 3-manifolds

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We will discuss new results on MatveevŠs complexity of infinite families of orientable hyperbolic 3-manifolds. We will demonstrate how TuraevŰViro invariants of hyperbolic 3-manifolds with totally geodesic boundary can be used to find complexity of manifolds.

The geometry of iterations defined on subsets of measure metric space and its applications to the tumor spread problem

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Researches devoted to cancer have occupied a key position in biology and medicine during last few decades. One of the most interesting problems related to this topic is connected to the cancer spread modeling. Usually model of tumor evolution is described as an iterative process. It means that there is an initial state of the tissue discribed as a set of ill and healthy cells, and each subsequent state is determined by the previous one. Moreover, it is assumed that the tumor spreads by a certain law and that the state of each cell is defined by the surrounding cells.

There are a large number of articles devoted to the discrete model of the cancer spread. Such models consider the tissue as $\mathbb{Z}^2$ and identify every element of $\mathbb{Z}^2$ with a cell. In the present talk we discuss more general model including the continuous case that describes the tissue as a metric space (X, d) with the cells corresponding to the points in this metric space. Suppose that there is a measure μ defined on the considered metric space, in fact, X is a measure metric space (mm-space). Consider a set $B_0 \subset X$ that relates to an initial configuration of the ill cells. The iterative process is determined by the following rule. Choose a real positive number r and some $k \in [0, 1)$. A point $x \in X$ becomes infected on the next step if the closed ball $B(x, r)$ with the center x and radius r contains at least k percent of the ill cells.

For the formal description of the iterative tumor evolution we introduce a function $P_r(A, x) = \frac{\mu(A \cap B(x,r))}{\mu(B(x,r))}$. Function $F(A) = \{x \in X : P_r(A, x) \geq k\}$ maps the set of all subsets of X to itself and characterizes the process of cancer spread. Tumor evolution with the initial state B_0 is described by the sets sequence $B_{i+1} = F(B_i)$. The first problem of the research is connected to properties desctiption of the mapping F and its iterations. For example, the stationary points of this mapping can be interpreted as the insistent tumors. We can define the notion of a curable tumor as a set B_0 such that $\mu(B_i) \to 0$ as $i \to \infty$. The second problem under consideration is classification of Borel sets in terms of curability. This talk will present several results connected with the model discussed above and describe some open-problems in this field.

The Drazin inverse for the sum of two matrices and representations

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We present some results concerning additive properties of the Drazin inverse for complex matrices. Furthermore, we give some representations for the Drazin inverse of a complex block matrix.

Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ with constant sectional curvature

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Nearly Kähler manifolds have been studies intensively in the 1970Šs by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is nonintegrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on $\mathbb{R}^7$. Recently it has been shown by Butruille that the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6- sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Totally real submanifolds are those for which the almost complex structure maps tangent vectors to normal vectors. A special case occurs when the dimension of the submanifold is half of the dimension of the ambiant space. In that case such submanifolds are called Lagrangian and J interchanges the tangent and the normal space. In this talk we study Lagrangian submanifolds of $S^3 \times S^3$. The first examples of such Lagrangian submanifolds were due to Schäfer and Smoczyk. Other examples have been recently discovered by Moroianu and Semmelmann. In this talk we obtain several classification results regarding such submanifolds. These results include the classification of the totally geodesic ones as well as the classification of the ones with constant sectional curvature.

Joint work with B. Dioos (Leuven) and X. Wang (Nankai).

Norm estimations for the Moore–Penrose inverse of multiplicative perturbations of matrices

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A multiplicative perturbation M of a matrix T is of the form $M = ETF^*$, where E and F are square matrices. This talk will focus on representations of the Moore-Penrose inverse $M^\dagger$, and some norm estimations for $M^\dagger - T^\dagger$. Some new ideals and techniques will be presented.

Concircularly Flat Z-symmetric Manifolds

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In the present paper, firstly, the definition of concircularly flat Z-symmetric manifold is given.

In the second section, the properties of this manifolds are examined.

This is joint work with Füsun Özen Zengin.

Some results on conformally flat almost contact metric manifolds

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In this talk which is based on a joint work with David E. Blair in the following reference, we first review some basic notions about almost contact metric manifolds. Next, we deal with conformally flat, ϕ-RK contact metric and almost contact metric manifolds. Finally, we focus on curvature properties of conformally flat generalized Sasakian space forms.

[1] Blair DE, Yıldırım H. On conformally flat almost contact metric manifolds. Mediterranean Journal of Mathematics. DOI:10.1007/s00009-015-0652-x.

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Fomenko invariants for the system: Chaplygin ball with a rotor

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We consider the problem of a rolling balanced dynamically nonsymmetric ball with a rotor on a rough horizontal plane. Earlier A.Y. Moskvin constructed bifurcation diagrams of the momentum mapping and bifurcation complexes in order to study the dynamics of the system and find the singular solutions. A natural continuation of this research is the fine Liouville analysis of the system. In this talk we present one of the steps in this direction, namely, we constract Fomenko invariants for this system and make rough topological analysis of the system.

Bier Complexes

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We introduce and study *Alexander r-tuples* $\mathcal{K} = \langle K_i \rangle_{i=1}^r$ of simplicial complexes, as a common generalization of pairs of Alexander dual complexes (Alexander 2-tuples) and r-unavoidable complexes of Blagojević, Frick, and Ziegler. In the same vein, the *Bier complexes*, defined as the deleted joins $\mathcal{K}^*_\Delta$ of Alexander r-tuples, include both standard *Bier spheres* and the so called *optimal multiple chessboard complexes* (introduced by Jojić, Vrećica, and Živaljević) as interesting, special cases.

Our main results are: (1) the r-fold deleted join of Alexander r-tuple is a pure complex homotopy equivalent to a wedge of spheres, and (2) the r-fold deleted join of a collectively unavoidable r-tuple is $(n-r-1)$-connected.

This research is a joint work with Gaiane Panina, Duško Jojić, and Ilya Nekrasov.

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