

On evolution of positively curved invariant Riemannian metrics on special Wallach spaces

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In the papers [2] and [3], the authors studied the normalized Ricci flow equation on one special class of Riemannian manifolds called generalized Wallach spaces (or three-locally-symmetric spaces in other terms) according to the definitions of [5] and [7]. Note that the complete classification of generalized Wallach spaces is obtained recently (independently) in the papers [4] and [6]. A given generalized Wallach space can be determined by special parameters $a_i \in (0, 1/2]$ (see details in [5]). Our main result is the following

Theorem 1. *On a generalized Wallach space with $a_1 = a_2 = a_3 := a = 1/4$ the volume normalized Ricci flow evolves all generic invariant Riemannian metrics into metrics with positive Ricci curvature.*

It should be noted that the case $a \in (0, 1/2] \setminus \{1/4\}$ was studied in [1].

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