Faculty of Sciences and Mathematics, University of Niš, Serbia
Faculty of Mathematics, University of Belgrade, Serbia
Faculty of Science, University of Kragujevac, Serbia
Mathematical Institute of the Serbian Academy of Sciences and Arts

XX GEOMETRICAL SEMINAR

Book of Abstracts

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Contents

PREFACE

The international conference 20th Geometrical Seminar 2018, is to be held at Vrnjačka Banja, Serbia from May 20th to 23rd, 2018.

This Book of abstracts is collection of abstracts of talks to be presented at the conference from different geometric topics. Participants will have 15 and 30 minutes talks. The 20th Geometrical Seminar has more than 120 participants from all over the world. This meeting is bringing together mathematicians, physicists and engineers interested in Geometry and its applications. The aim of the Geometrical Seminar is to enable researches to give lectures on new results, exchange ideas, problems and conjectures. This issue contains abstracts of talks to be presented at the 20th Geometrical Seminar. The abstracts were accepted for presentation after having been subjected to the usual strict reviewing process of the conference committee. The editors thank the members of the Committees of Geometrical Seminar and to the Faculty of Sciences and Mathematics, Niš, Faculty of Mathematics, Belgrade, to the Faculty of Science, Kragujevac and Mathematical Institute SANU for their great effort in organizing this conference.

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Osserman, Clifford, and duality properties

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We consider pseudo-Riemannian generalizations of Osserman, Clifford, and the duality principle properties for algebraic curvature tensors and investigate relations between them.

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Topological surgery in nature

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Topological surgery is a mathematical technique used for creating new manifolds out of known ones. We observe that it occurs in natural phenomena where forces are applied and the manifold in which t hey occur changes type. For example, 1-dimensional surgery happens during chromosomal crossover, DNA recombination and when cosmic magnetic lines reconnect, while 2-dimensional surgery happens in the formation of Falaco solitns, in drop coalescence and in the cell mitosis. Inspired by such phenomena, we enhance topological surgery with the observed *forces* and *dynamics*. We then generalize these low-dimensional cases to a model which extends the formal definition to a continuous process caused by local forces for an arbitrary dimension m. Next, for modelling phenomena which do not happen on arcs, respectively surfaces, but are 2-dimensional, respectively 3-dimensional, we fill in the interior space by defining the notion of *solid topological surgery*. We further present a dynamical system as a model for both natural phenomena exhibiting a 'hole drilling'behavior and our enhanced notion of solid 2-dimensional 0-surgery. Moreover, we analyze the ambient space S^3 in order to introduce the notion of *embedded topological surgery* in S^3. This notion is then used for modelling phenomena which involve more intrinsically the ambient space, such as the appearance of knotting in DNA and phenomena where the causes and effects of the process lie beyond the initial manifold, such as the formation of tornadoes. Moreover, we present a visualization of the 4-dimensional process of 3-dimensional surgery by using the new notion of *decompactified 2-dimensional surgery* and *rotations* and propose a model for a phenomenon exhibiting 3-dimensional surgery: the formation of black

holes from cosmic strings. Finally, we propose of connection of our model with Morse theory. We hope that through this study, topology and dynamics of many natural phenomena, as well as topological surgery itself, will be better understood.

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Homogeneous geodesics and two-step homogeneous geodesics in homogeneous spaces

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Let $(M = G/H, g)$ be a homogeneous Riemannian manifold. A geodesic $\gamma(t)$ through $o = eK$ is called *homogeneous* if it is an orbit of a 1-parameter subgroup G, i.e. $\gamma(t) = \exp tX \cdot o$, $0 \neq X \in \mathfrak{g}$ (X = geodesic vector). Then $M = G/K$ is called *g.o. space* (or space with homogeneous geodesics) if any geodesic γ of M is homogeneous. A Riemannian manifold (M, g) is called *g.o. manifold* (or a manifold with homogeneous geodesics) if any geodesic γ of M is an orbit of a 1-parameter subgroup of the full isometry group of (M, g). Recently we initiated the study of geodesics of the form $\gamma(t) = \exp tX \exp tY \cdot o, \quad X, Y \in \mathfrak{g}$ in a homogeneous space G/K, which we call *two-step homogeneous geodesics*. In the present talk I will present some new results related to homogeneous geodesics and two-step homogeneous spaces in various homogeneous spaces, based on joint works with Y. Wang and Z. Zhao, N.P. Souris and G. Calvaruso.

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On the (m, ρ)-quasi Einstein manifolds and Ricci solitons

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The m-Bakry-Emery Ricci tensor, which is closely related to the Ricci solitons and is a natural generalization of Ricci tensor, was introduced and many geometers have described various special manifolds, in which the m-Bakry- Emery Ricci tensor is proportional to metric tensor. One of these special manifolds is the (m, ρ)-quasi Einstein manifold.

In this paper, the authors consider (m, ρ)-quasi Einstein manifolds. It is researched that in which conditions the Ricci soliton structure can be observed on these manifolds. Then, an example for this kind of manifolds is given in the following part of the study.

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Left invariant geometry of complex hyperbolic plane

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In this talk we consider complex hyperbolic plane as a metric Lie group. We give a classification of nonisometric left invariant Riemannian metrics and left invariant Hermitian structures on Lie group which is the noncompact part in the Iwasawa decomposition of unitary group $SU(1,2)$. This group is completely solvable and acts simply transitively on complex hyperbolic plane. We examine the geometry of this group equipped with left invariant metrics: curvature properties, self-duality and holonomy.

AMS Subj. Class.: 53B35, 22E60, 53C20 Key words: complex hyperbolic space, left invariant metrics, Hermitian complex structures

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The applications of canonical structures on generalized symmetric spaces

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Generalized symmetric spaces (in particular, homogeneous k-symmetric spaces) G/H admit a commutative algebra $\mathcal{A}(\theta)$ of canonical structures. The remarkable feature of these structures is that all of them are invariant with respect to both the Lie group G and the generalized s̈ymmetriesöf G/H. The classical example is the canonical almost complex structure J on homogeneous 3-symmetric spaces with its many applications (N.A.Stepanov, A.Ledger, A.Gray, J.A.Wolf, V.F.Kirichenko, S.Salamon and others). For $k > 3$ the algebra $\mathcal{A}(\theta)$ contains a large family of classical structures such as almost complex ($J^2 = -id$), almost product ($P^2 = id$), f-structures of K.Yano ($f^3 + f = 0$) and some others (the author and N.A.Stepanov). It turned out that the canonical structures J, f and P provided a wealth of invariant examples for several directions: the generalized Hermitian geometry, specifically, metric f-structures (V.F.Kirichenko, D.Blair, C.J.C.Negreiros, L.A.B.San Martin, the author, A.Sakovich, A.Samsonov and others), homogeneous Riemannian geometry (e.g., for the Naveira classification of Riemannian almost product structures), geometry of elliptic integrable systems (F.Burstall, I.Khemar). Particularly, it was obtained a significant analogy with some classical results in Hermitian geometry and their generalizations. Here we continue this topic and concentrate on two recent applications of canonical structures.

Left-invariant structures on Lie groups. Using the theory of canonical structures, we (joint with P.Dubovik and O.Radivanovich) present a number of left-invariant nearly Kähler and Hermitian f-structures on some

classes of nilpotent Lie groups (especially, on 2-step nilpotent and some filiform Lie groups). We also characterize the corresponding left-invariant canonical distributions with respect to the classes **F** (foliations), **AF** (anti-foliations), **TGF** (totally geodesic foliations). In particular, several generalized (in various senses) Heisenberg groups were considered in more detail. They are the 5-dimensional Heisenberg group $H(2,1)$, the 6-dimensional generalized (in the sense of A.Kaplan) Heisenberg group, the 8-dimensional block-matrix generalized Heisenberg group.

Metallic structures on manifolds. We also consider arbitrary linear deformations of almost product structures on smooth manifolds. In particular cases this approach leads to so-called metallic structures (golden, silver and others), which are fairly popular (especially, golden structures) in many recent publications (M.Crasmareanu, C.-E.Hretcanu, A.Salimov, F.Etayo and others). The general result on the integrability of linear deformations was proved. This covers previous particular cases for all metallic structures. Finally, all canonical structures of metallic family on homogeneous k-symmetric spaces have been completely described.

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Einstein submanifolds of the total space of the cotangent bundle

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Our purpose is to provide here a family of examples of Einstein manifolds with positive scalar curvature, constructed as submanifolds of the total space of the cotangent bundle. The presentation is based on [Bejan, C. L.; Eken, S.; Kilic, E. - Adv. Appl. Clifford Algebras (2017)].

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On canonical almost geodesic mappings of type π_2(e)

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In this study we consider canonical almost geodesic mappings of type $\pi_2(e)$. We have found the conditions that must be satisfied for the mappings to preserve the Riemann tensor. Also we consider canonical almost geodesic mappings of type $\pi_2(e)$ of spaces with affine connections onto symmetric spaces. The main equations for the mappings are obtained as a closed mixed Cauchy type system of PDEs. We have found the maximum number of essential parameters on which the solution of the system depends.

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Differential geometry and representations of semisimple algebraic groups

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The results of this work were obtained in collaboration with V.V. Lychagin. In this talk we discuss an approach to the study of orbits of actions of semisimple algebraic groups in their irreducible complex representations, which is based on differential invariants on the one hand, and on geometry of reductive homogeneous spaces on the other hand.

We start from the well-known problem of SL_2-classification of binary forms, which was studied by many famous mathematicians during XIX and XX centuries. Classical invariant theory starts from this problem. But it appears that the full solution of this problem can be obtained with the help of differential equations and differential invariants. Namely, we represent each binary form of degree n as a solution of the so–called Euler equation $xu_x + yu_y = nu$, and study differential invariants for the SL_2-actions on the prolongations of this equation. We prove that the dependence between basic differential invariant and its derivations uniquely defines the SL_2-orbit of a given binary form. We also present some examples.

It the second part of the talk we generalize this approach for the problem of classification of G-orbits of a given connected semisimple algebraic group G in its irreducible representation. According to the Borel-Weil-Bott theorem, every irreducible representation of connected semisimple Lie group is

isomorphic to the action of this group on the module of holomorphic sections of some one-dimensional bundle over the flag variety G/B. Using this, we give a complete description of the structure of the field of differential invariants for this action and obtain a criterion which separates regular G-orbits.

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Extensibility of geodesics in Gromov–Hausdorff space

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We discuss geodesics in Gromov–Hausdorff space. It is known that Gromov–Hausdorff space $\mathcal{M}$ is a geodesic metric space and that $\forall X, Y \in \mathcal{M}$ $d_{GH}(X, Y) \leq \frac{1}{2}\max(\operatorname{diam}(X), \operatorname{diam}(Y))$. We deal with the case, when $\operatorname{diam}(X) \leq \operatorname{diam}(Y)$ and $d_{GH}(X, Y) = \frac{1}{2}\operatorname{diam}(Y)$. For such X and Y we study conditions under which a geodesic between X and Y cannot be obtained as a restriction of some other geodesic between X and some $Z \in \mathcal{M}$. For cases when these conditions are not met we provide examples of geodesics that can be obtained as a restriction.

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Steiner minimal trees in small neighbourhoods of points in Riemannian manifolds

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In contrast to the Euclidean case, almost no Steiner minimal trees with concrete boundaries on Riemannian manifolds are known. A result describing the types of Steiner minimal trees on a Riemannian manifold for arbitrary small boundaries is obtained. As a consequence, it is shown that for sufficiently small regular n-gons with $n > 7$ their boundaries without a longest side are Steiner minimal trees.

To prove the main theorem we looked at variations of metrics. We pointed out that the fact that the lengths of curves vary continuously with the metric does not imply that the distances between points in the corresponding intrinsic metric also vary continuously; the relevant example was presented. The following question have to be considered: what should we assume to ensure that the distance varies continuously? Sufficient condition was formulated.

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Application of Shape Operator Under Infinitesimal Bending of Surface

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In case of bendable surfaces it is useful to discuss the variation of magnitudes such as the shape operator. The shape operator is a good way to measure how a regular surface S bends in $\mathcal{R}^3$ by valuation how the surface normal ν changes from point to point. We considered the variation of shape operator under infinitesimal bending of surface given in an explicit form and its application in considering what happened with the elliptic, hyperbolic, parabolic kind of points under the infinitesimal bending of surface.

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Key words: Shape operator, Infinitesimal bending, Variation, Elliptic, Hyperbolic and Parabolic points.

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On the dual quaternionic inclined curves

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In this study, a brief summary about dual quaternions and dual quaternionic curves are firstly presented. Also, the definition of inclined curve and harmonic curvature is given. Secondly, we describe dual quaternionic inclined curves and harmonic curvatures for the dual quaternionic curves. Moreover, we give characterizations for a dual quaternionic curve to be a dual quaternionic inclined curve. Finally, we obtain some characterizations for the dual quaternionic inclined curves in terms of the harmonic curvatures.

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Some classification results for low-type curvature-adapted hypersurfaces of quaternionic space forms

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In a curvature-adapted hypersurface M of a quaternionic- Kähler manifold $\overline{M}$ the maximal quaternionic subbundle $\mathcal{D}$ of TM and its orthogonal complement $\mathcal{D}^{\perp}$ in TM are, by definition, invariant subspaces of the shape operator A at each point. We classify curvature-adapted real hypersurfaces M of non-flat quaternionic space forms $\mathbf{H}P^m$ and $\mathbf{H}H^m$ that are of Chen 2-type in an appropriately defined (pseudo) Euclidean space of quaternion-Hermitian (symplectic) matrices. That means that the position vector of such submanifold in the ambient (pseudo) Euclidean space is decomposable into a sum of a constant vector and two nonconstant vector eigenfunctions of the Laplace operator of the submanifold, belonging to different eigenspaces. In the quaternionic projective space they include all geodesic hyperspheres except one, two series of tubes about canonically embedded quaternionic projective spaces of lower dimensions and two particular tubes about canonically embedded $\mathbf{C}P^m \subset \mathbf{H}P^m$. Except for these two tubes, other tubes about $\mathbf{C}P^m$ are mass-symmetric and of 3-type.

On the other hand, the list of 2-type curvature-adapted hypersurfaces in $\mathbf{H}H^m$ is reduced to geodesic spheres and tubes of arbitrary radius about totally geodesic quaternionic hyperplane $\mathbf{H}H^{m-1}$.

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Nonolocal gravity cosmologial solutions and perturbations

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After discovery of accelerating expansion of the Universe, there has been a renewed interest in gravity modification. One of promising approaches is nonlocal modification with the scalar curvature R in the action replaced by a suitable function $F(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is the Laplace-Beltrami operator. In particular we analyze the modification in the form

$$S = \int \sqrt{-g}\Big(\frac{R}{16\pi G} + P\mathcal{F}(\Box)Q\Big)d^4x,$$

where $\mathcal{F}(\Box)$ is an analytic function. Also we will discuss perturbations of this model as a background.

This is joint work with Branko Dragovich, Jelena Stankovic and Zoran Rakic.

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Distribution of the angles associated to indefinite integral binary quadratic forms

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To each indefinite integral binary quadratic form Q, we may associate the geodesic in hyperbolic plane $\mathbb{H}$ through the roots of quadratic equation $Q(x, 1)$. We study the asymptotic distribution (as discriminant of quadratic form Q tends to infinity) of the angles between these geodesics and one fixed geodesic which intersects all of them.

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Geodesic lines on nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$

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A nearly Kähler manifold (NK) is an almost Hermitian manifold (M, g, J) for which the tensor ∇J is skew-symmetric: $(\nabla_X J)Y + (\nabla_Y J)X = 0$, for all $X, Y \in TM$, where ∇ is the Levi-Civita connection of the metric g. The first example of such manifolds was introduced on $\mathbb{S}^6$ by Fukami and Ishihara and, later, these manifolds have been intensively studied by A. Gray. More recently interest in NK manifolds increased because these manifolds are examples of geometries with torsion and therefore they have applications in mathematical physics. The case of 6-dimensional NK manifolds is of particular importance because of several recent results, mostly by P-A. Nagy and J-B. Butruille. Moreover, six-dimensional NK manifolds are Einstein and are related to the existence of a Killing spinor, which inspires their further investigation. It was shown that there are only four different NK homogeneous manifolds of dimension 6; one of them is $\mathbb{S}^3 \times \mathbb{S}^3$, with a metric different than a standard euclidean product metric. An important tool in the study of NK $\mathbb{S}^3 \times \mathbb{S}^3$ is the use of an almost product structure P. In order to classify some types of submanifolds of NK $\mathbb{S}^3 \times \mathbb{S}^3$, it would be interesting to know how geodesic lines look like. In our joint work we obtained explicit formulas for geodesic lines on NK $\mathbb{S}^3 \times \mathbb{S}^3$, as well as some of their properties. This is joint work with Mirjana Djorić and Marilena Moruz.

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The symmetrization of jets on vector bundle

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Roughly speaking, we solve some existence problems of holonomic prolongation of connections and symmetrization of semiholonomic jets. We recall that higher order jets are a very powerful tool in differential geometry and in many areas of mathematical physics. For example, holonomic jets globalize the theory of differential systems and semiholonomic jets play an important role in the calculus of variations and in the theory of PDE's. But the most important role in differential geometry and also in mathematical physics is played by classical holonomic jets. This leads to the problem of symmetrization of r-th order semiholonomic jets on vector bundles. More precisely, we solve the following problem.

Problem. *Under which conditions there is a canonical fibered map* $S_E : \overline{J}^r E \to J^r E$ *with* $S_{E|J^r E} = id_{J^r E}$, *where* $\overline{J}^r E$ *or* $J^r E$ *is an* r*-th order semiholonomic or holonomic prolongation of a vector bundle* $E \to M$, *respectively.*

We show that this problem is very closely connected with the existence of holonomic prolongation of general linear connections.

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The natural Lie algebra brackets on couples of vector fields and p-forms

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Let $\mathcal{M}f_m$ be the category of m-dimensional $\mathcal{C}^\infty$ manifolds and their embeddings.

The "doubled" tangent bundle $T\oplus T^*$ over $\mathcal{M}f_m$ is full of interest because of it has the Courant bracket, see [1]. Besides, generalized complex structures are defined on $T \oplus T^*$, generalizing both (usual) complex and symplectic structures, see e.g. [4], [5]. In [5], Hitchin extended the Courant bracket to the generalized one on $T \oplus \bigwedge^p T^*$ for any p.

In [3], if $m \geq p+1 \geq 2$ or $m = p \geq 3$, we classified all $\mathcal{M}f_m$-natural bilinear operators $A : (T \oplus \bigwedge^p T^*) \times (T \oplus \bigwedge^p T^*) \rightsquigarrow T \oplus \bigwedge^p T^*$ transforming pairs of couples $X^i \oplus \omega^i \in \mathcal{X}(M) \oplus \Omega^p(M)$ $(i = 1, 2)$ of vector fields and p-forms on m-manifolds M into couples $A(X^1 \oplus \omega^1, X^2 \oplus \omega^2) \in \mathcal{X}(M) \oplus \Omega^p(M)$ of vector fields and p-forms on M. In particular, we proved that if $m \geq p+1 \geq 2$ (or $m = p \geq 3$, respectively), then any $\mathcal{M}f_m$-natural skew-symmetric bilinear operator $A : (T \oplus \bigwedge^p T^*) \times (T \oplus \bigwedge^p T^*) \rightsquigarrow T \oplus \bigwedge^p T*$ coincides with the Courant bracket up to three (or two, respectively) real constants.

In earlier work [2], we obtained the above results only for $m \geq 2$ and $p = 1$. Moreover, if $m \geq 2$ we found all $\mathcal{M}f_m$-natural bilinear operators

$A : (T \oplus T^*) \times (T \oplus T^*) \rightsquigarrow T \oplus T^*$ satisfying the Leibniz rule, and we found all $\mathcal{M}f_m$-natural Lie algebra brackets $[-,-]$ on $\mathcal{X}(M) \oplus \Omega^1(M)$.

In the present paper, using the results of [3], if $m \geq p+1 \geq 2$ or $m = p \geq 3$, we find all $\mathcal{M}f_m$-natural bilinear operators $A : (T \oplus \bigwedge^p T^*) \times (T \oplus \bigwedge^p T^*) \rightsquigarrow T \oplus \bigwedge^p T^*$ satisfying the Leibniz rule

$$A(X, A(Y,Z)) = A(A(X,Y),Z) + A(Y, A(X,Z))$$

for any $X, Y, Z \in \mathcal{X}(M) \oplus \Omega^p(M)$ and $M \in obj(\mathcal{M}f_m)$, and then we find all $\mathcal{M}f_m$-natural Lie algebra brackets $[-,-]$ on $\mathcal{X}(M) \oplus \Omega^p(M)$ (i.e. $\mathcal{M}f_m$-natural skew-symmetric bilinear operators $A = [-,-]$ as above satisfying the Leibniz rule).

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p-Adic distance and structure of the genetic code

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p-Adic distance (p is a prime number) is the most employed example of ultrametrics, which satisfies strong triangle inequality,

$$d(x, y) \leq max\{d(x, z), d(y, z)\}.$$

It plays a central role in p-adic analysis and its applications in modeling physical and biological systems with hierarchical structure (for a recent review, see [1]). Due to its ultrametricity, p-adic distance, induced by p-adic norm, has some unusual properties, which appear to be natural in many applications. For example, it is related to divisibility of the difference of two integers with respect to prime number p: smaller p-adic distance – larger divisibility. In a sense, p-adic distance is related to similarity of two numbers.

The genetic code is a map from the set of 64 codons onto the set of 20 amino acids and one stop signal. It is almost unique in all living organisms with respect to a huge number of mathematical possibilities. The vertebrate mitochondrial (VM) code is relatively simple and other dozens genetic codes can be considered as its slight variations. In the VM code, an amino acid is coded by one, two or three codon doublets. When a codon doublet code the same amino acid, one can say that these codons are close, or similar, in the informational sense.

In a few papers (see [2] and references therein) is shown that the p-adic distance is simple and adequate mathematical tool to describe codon

closeness (similarity). Modeling the VM code, we assign 64 natural numbers in the form $a_0+a_15+a_25^2$ to the 64 codons identifying nucleotides in codons with digits a_i as follows: C (Cytosine) = 1 , A (Adenine) = 2, U (Uracil) = T (Thymine) = 3, and G (Guanine) = 4. With respect to the smallest 5-adic distance between codons one obtains 16 quadruplets. These quadruplets split into two doublets under 2-adic distance. Each of these 32 doublets codes an amino acid or stop signal. This p-adic approach has been extended to many other aspects of the genetic code and bioinformation. This is a review talk with some new developments.

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Contact magnetic curves in Sol space

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Magnetic curves represent trajectories of charged particles moving on a Riemannian manifold under the action of a magnetic field.

Among the eight 3D homogeneous model spaces, S^3, Nil_3, $\widetilde{\mathrm{SL}}_2\mathrm{R}$ and Sol_3 admits a contact structure compatible to the metric. The compatible contact structure naturally induces a magnetic field on these four model spaces.

In this talk we consider magnetic curves with respect to the canonical contact structure of the Sol space.

The study of magnetic curves in arbitrary Riemannian manifolds was developed in early 1990's, even though related works can be found earlier (see [4, 7]). Recently there are interesting results on magnetic curves in Euclidean space [5], Sol space [3], Sasakian manifolds [1], cosymplectic manifolds [2] and on a 3-torus [6].

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Change of monomial bases in Steenrod algebra mod p

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I shell talk about some new bases in Steenrod algebra $\mathcal{A}_p$ over $\mathbb{Z}/p$. I shell give a few constructions of monomial bases and then focus on the next problem: Whether the transition matrix between two fixed bases is triangle. This problem is important for calculation in Steenrod algebra.

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Cobordisms of graphs. Sliceness criterion for odd graphs

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Cobordism in 1-dimensional case is a well-known and long-studied equivalence relation on classical knots. In a natural way it can be defined for virtual knots, and by the standard procedure involving Gauss diagrams — for free knots. In turn, free knots may be interpreted as framed 4-valent graphs. Thus, a notion of *framed 4-graphs cobordism* arises. we consider graphs with one unicursal component.

To be precise, we give the following definition:

Definition. *Two framed graphs* Γ_1, Γ_2 *are said to be* cobordant *if there exists a triple* (M^3, S, f) *where* M^3 *is a 3-manifold,* S_g *is an orientable 2-surface and* $f\colon S_g \to M^3$ *such that*

- *the image of* f *has only standard types of singularities (double lines, triple points, cusp points);*
- $f(\partial(S_g \setminus (D^2 \sqcup D^2)) = \Gamma_1 \sqcup \Gamma_2$.

Minimal g *among the surfaces* S_g *in such triples is called the* cobordism genus.

Definition. *A framed 4graph* Γ *is called* slice *if it is cobordant to the trivial graph given by a circle with no vertices.*

The question of sliceness for graphs is interesting and important. Recently the following sliceness criterion was proved (together with V.O. Manturov):

Theorem. *Let* Γ *be framed 4-graph such that every vertex of* Γ *is odd in the sense of Gaussian parity. Then* Γ *is genus zero slice if and only if there exists a pairing of the chords of its chord diagram without intersections.*

This theorem is important for in case of odd framed 4-graphs it gives a finite combinatorial algorithm solving the problem of sliceness.

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Surfaces with parallel normalized mean curvature vector field in 4-spaces

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A basic class of surfaces in Riemannian and pseudo-Riemannian geometry are the surfaces with parallel mean curvature vector field, since they are critical points of some natural functionals and play important role in differential geometry, the theory of harmonic maps, as well as in physics. A natural extension of the class of surfaces with parallel mean curvature vector field are surfaces with parallel normalized mean curvature vector field.

On any surface with parallel normalized mean curvature vector field in a pseudo-Euclidean 4-space we introduce special geometric (canonical) parameters that allow us to describe this class of surfaces in terms of three invariant functions satisfying a system of three partial differential equations.

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Delaunay type surfaces and curvature extremal curves

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We consider Constant Mean Curvature (CMC) rotational surfaces in Riemannian 3-space forms $M^3(c)$. We see that, locally, they are swept out by extremal curves of a curvature energy whose simplest form in $\mathbb{R}^3$ was studied by Blaschke [2, 3]. These extremals are explicitly parametrized and have an associate Killing vector field what allows us to see this family of surfaces as Binormal evolution surfaces in $M^3(c)$ [1]. Using these facts a complete local classification of CMC surfaces is given, recovering, in a unified manner, various well known classification results on the subject. Finally, compactness and completeness are analyzed in terms of the topological properties of the profile curves.

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Fiberwise two-sided multiplications on homogeneous C^*-algebras

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We consider fiberwise two-sided multiplications on algebras of sections of locally trivial complex $n \times n$ ($n \geq 2$) matrix bundles over metrizable compact spaces (so called n-homogeneous C^*-algebras). Using algebraic topology language, we give necessary and sufficient conditions when are such maps automatically global two-sided multiplications. As a consequence, we get that on such algebras non-global fiberwise two-sided multiplications always exist whenever the base space contains a nonempty open subset homeomorphic to (an open subset of) $\mathbb{R}^d$ for some $d \geq 3$. This is a joint work with R. M. Timoney (Trinity College Dublin).

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On global conformal mappings onto Einstein spaces

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We obtain (with J. Mikes and I. Hinterleitner) new conditions on (pseudo-) Riemannian spaces of conformal mappings onto Einstein spaces which this mappings are homothetic.

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Symmetries in 4-dimensional manifolds

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In this talk I will discuss some algebraic and geometrical properties of symmetries (taken here as Lie algebras of smooth Killing vector fields) on a 4−dimensional manifold admitting a metric of arbitrary signature. The discussion will include the theory of the distributions arising from such vector fields, their resulting orbit and isotropy structures and certain topological and stability properties which these orbits may, or may not, possess. This will involve a survey of the subalgebras of the Lie algebras o(4), o(1,3) and o(2,2). A link between the isotropies and the restrictions on the fundamental tensors of Ricci and Weyl will be indicated and some examples supplied.

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On global conformal mappings onto Einstein spaces

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We obtain (with N. Guseva and J. Mikes) new conditions on (pseudo-) Riemannian spaces of conformal mappings onto Einstein spaces which this mappings are homothetic.

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Quaternionic Heisenberg group and Solutions to Strominger system with non-constant dilaton in dimensions 7, 6 and 5

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Solutions to the Strominger system (generalized heterotic string version of the Einstein vacuum equations of general relativity) with non trivial fluxes is presented. These supersymmetric solutions solve also the heterotic equations of motion up to the first order of the string tension α.

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Virtual knots and knotoids

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Virtual knot theory studies the abstract properties of knot diagrams that can represent knots and links in thickened surfaces. There is a diagrammatic theory of virtual knots, using planar knot and link diagrams augmented with virtual crossings and moves that extend the Reidemeister moves. Two virtual diagrams are equivalent by these moves if and only if corresponding embeddings in thickened surfaces are stable equivalent (equivalent up to the addition and subtraction of surface 1-handles that are disjoint from the knot). There are many remarkable phenomena in virtual knot theory and new invariants of great interest. This talk will discuss basics of virtual knot theory and how it can be applied to the study of knotoids. Knotoids are classical knot diagrams with free ends and are taken up to Reidemeister moves that do not slide arcs across these free ends. Knotoids are also of great interest and can be used in applications to model the topological structure of long chain molecules. We will also discuss virtual knotoids and other variants of virtual theory with a texture of examples and particular invariants.

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Integrable billiard's books

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2000 Mathematics Subject Classification: 70H99

A billiard's book is a new integrable Hamiltonian system that extends to the case of the billiard in a domain bounded by confocal quadrics. Such type of billiards formed by gluing a few classical billiard domains along pieces of their boundaries. The special case where we glue two domains called a topological billiard and was researched by V.V. Fokicheva [1].

The billiard's book dynamical system has 4-dimensional phase space and two integrals: the scalar square of the velocity vector and one special integral. The second special integral can be described by the following feature of trajectories: the straight lines containing the segments of the polygonal billiard trajectory are tangents to a certain quadric (ellipse or hyperbola).

Integrable Hamiltonian systems with 2 degrees of freedom have Fomenko-Zieschang invariants [2]. Such invariants allow us to speak about the equivalence between closures of trajectories.

Researching billiard's books we try model famous integrable systems in terms of Fomenko-Zieschang invariants. The Fomenko conjecture about modeling Fomenko-Zieschang invariants using billiard's books and new results that confirm the part of the conjecture will be presented.

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2000 Mathematics Subject Classification: 70H99

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On Bertrand curves in Minkowski 3-space

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In this talk, we study spacelike and timelike Bertrand curves in Minkowski 3-space. According to the casual character of the principal normal vector, we show that the Bertrand mate curve can be spacelike, timelike or null curve. Thus, we give the necessary and sufficient conditions for spacelike and timelike curves to be Bertrand curves and we also give the related examples.

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On some applications of Killing symmetries in 4-dimensional manifolds admitting a metric

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The purpose of this work is to investigate Killing symmetries on 4–dimensional smooth, connected manifolds admitting a metric of arbitrary signature. Firstly, some basic notions about the Lie algebra of Killing vector fields are introduced on these manifolds. After that several examples are given when the metric signature is $(+,+,-,-)$ (referred to as neutral signature). For each of these examples, the dimensions of the Lie algebra of Killing vector fields and nature of the Killing orbits are determined and the isotropy subalgebras are computed. The algebraic natures of the Weyl and Ricci tensors are then studied since they depend crucially on the isotropy subalgebra and this is illustrated with examples. Finally, some results and remarks regarding Killing symmetries are presented for Lorentz and positive definite signatures.

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Convexity of balls in Gromov-Hausdorff space

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In this paper we study the space M of all nonempty compact metric spaces considered up to isometry, equipped with the Gromov-Hausdorff distance. We show that each ball in M with the center at the one-point space is convex in the weak sense, i.e., every two points of such a ball can be joined by a shortest curve that belongs to this ball, and not convex in the strong sense: it is not true that every shortest curve joining the points of the ball belongs to this ball. It is also shown that a ball of sufficiently small radius with the center at a space of general position is convex in the weak sense.

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Mathematical Conference: XX Geometrical Seminar
May 20 - 23, 2018, Vrnjačka Banja, Serbia

On geometric measure theory and submanifolds

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A. Gray and L. Vanhecke proposed the "Riemannian volume conjecture" which says the following: On an analytic Riemannian manifold M, if the volume of each sufficiently small geodesic ball is the same as the volume of the corresponding Euclidean ball, then M is locally Euclidean. (Acta Math. 1979). The subsequent paper by O. Kowalski (Acta Math. 1980) casted doubt on the validity of this conjecture. This point of view was, after a long time, supported by Chinatsu Ueda (Tokyo J. Math.,2000). Yet, the conjecture remains still undecided.

In our lecture, we concentrate on the extrinsic version of the previous problem. We consider a special Riemannian manifold, namely a hypersurface Mn of the Euclidean space R^{n+1}. For any $m \in M$, we denote by $B(m,r)$ the Euclidean ball in R^{n+1} with the center m and radius r. Further,let us denote by α_n the volume of the unit Euclidean ball in R_n.

We classify explicitly all hypersurfaces M with the following property: The Riemannian volume $Vol(B(m,r) \cap M)$ is equal to $\alpha_n r^n$ for every $r > 0$. The basic examples of such hypersurfaces are a) hyperplanes. b) the direct product of the form $R^{n-3} \times C_1^3$, where C_1^3 is the "light cone" $x_4^2 = x_1^2 + x_2^2 + x_3^2$ in R^4 .

The pioneering work by David Preiss in the geometric Measure Theory was used here as the main argument for $r \to \infty$.

It was combined with a differential geometric argument valid for $r \to 0$.

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Superintegrable Bertrand natural mechanical systems

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The problem of description of superintegrable systems (i.e., systems with closed trajectories in a certain domain) in the class of spherically symmetric natural mechanical systems goes back to Bertrand and Darboux. Dynamical and geometric properties of such systems have been studied by many mathematicians (Killing, Besse, Perlick, Kozlov, Borisov, Mamaev, Santoprete, Ballesteros, Enciso, Herranz, Ragnisco). However in full generality, the problem of description of all such systems remained open because of the so-called "problem of equators". Let us proceed with precise statements.

Bertrand proved [1] that, in Newtonian mechanics, the Kepler potential $V(r) = -a/r + b$ and the harmonic oscillator potential $V(r) = -ar^2 + b$ $(a, r \in \mathbb{R},\ a > 0)$ are distinguished by the property that all bounded trajectories are periodic and there exists at least one non-circular periodic trajectory. Natural mechanical systems possessing the above property will be called Bertrand systems. Perlick [2] obtained a complete description of all spherically symmetric Bertrand systems, whose underlying Riemannian manifolds of revolution have no equators [3].

We give a complete description of all spherically-symmetric Bertrand systems [4]. In particualar, we do not assume that the underlying Riemannian manifold of revolution has no equators.

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On knotoids, braidoids and applications

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The theory of knotoids, which are open curves in oriented surfaces, is introduced by Turaev and it is a non-trivial extension of classical knot theory. Knotoids were further studied by Bartholomew and by Gügümcü and Kauffman. Recently, Gügümcü and Lambropoulou introduced the counterpart theory of braidoids, which extends the classical theory of braids, and their topological interaction with knotoids. The theory of knotoids has also found important applications in the topological analysis of proteins (Goundaroulis, Dorier, Benedetti, Stasiak and Goundaroulis, Gügümcü, Lambropoulou, Dorier, Stasiak, Kauffman). In this talk we will present some aspects of the theory of knotoids and of braidoids and we shall also discuss applications.

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Novikov fundamental group

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Novikov homology has been introduced by Novikov in the early 1980s motivated by problems in hydrodynamics. The Novikov inequalities in the Novikov homology theory give lower bounds for the number of critical points of a Morse closed 1-form on a compact differentiable manifold M. In the first part of my talk I shall survey the Novikov homology theory in finite dimensional setting and its further developments in infinite dimensional setting with applications in the theory of symplectic fixed points and Lagrangian intersection/embedding problems. In the second part of my talk I shall report on my recent joint work with Jean- Francois Barraud and Agnes Gadbled on construction of the Novikov fundamental group associated to a cohomology class of a closed 1-form on M and its application to obtaining new lower bounds for the number of critical points of a Morse 1-form.

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On polyhedral products over 2-truncated cubes

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In this talk we are going to discuss higher Massey products in cohomology and rational formality of (generalized) moment-angle manifolds $\mathcal{Z}_P$ where P is a 2- truncated cube, that is, a consecutive cut of only codimension-2 faces starting with a cube. Namely, we introduce a family of n-dimensional 2-truncated cubes P, one for each $n \geq 2$, which starts with a square, such that there is a defined, nontrivial and uniquely determined n-fold Massey product in cohomology of $\mathcal{Z}_P$ for any $n \geq 2$. Moreover, we introduce a family of (generalized) moment-angle manifolds over 2- truncated cubes which contains a manifold with prescribed connectedness and order of a nontrivial higher Massey product (containing a unique element) in cohomology. Therefore, these manifolds are not formal spaces. On the other hand, we prove a combinatorial criterion for a simple graph Γ to provide a rationally formal (generalized) moment-angle manifold $\mathcal{Z}_P$ over a graph- associahedron $P = P_\Gamma$.

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Metrics transformations preserving the types of one-dimensional minimal fillings

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Let M be an arbitrary finite set and $G = (V, E)$ be a connected graph. We say that G *joins* M, and M is *the boundary of the graph* G if $M \subset V$. The boundary of a graph G is denoted by ∂G. Now let $\mathcal{M} = (M, \rho)$ be a finite pseudo-metric space, $G = (V, E)$ be a connected graph joining M, and $\omega : E \to \mathbb{R}_+$ be a mapping into nonneggative real numbers usually called a *weight function* generating the *weighted graph* $\mathcal{G} = (G, \omega)$. The *weight* of a weighted graph $\mathcal{G}$ is the value $\omega(G)$ equal to the sum of weights of all edges of the graph. The function ω generates a pseudo-metric d_ω at V, namely, the distance between vertices of the graph $\mathcal{G}$ is the least weight of paths joining those vertices. If for any points p and q from M the inequality $\rho(p, q) \leq d_\omega(p, q)$ holds, then the weighted graph $\mathcal{G}$ is called a *filling* of the space $\mathcal{M}$ and the graph G is called the *type of this filling*. The number $\mathrm{mf}(\mathcal{M}) = \inf \omega(\mathcal{G})$, where the infimum is taken over all fillings $\mathcal{G}$ of the space $\mathcal{M}$, is called the *weight of minimal filling*, and the filling $\mathcal{G}$ such that $\omega(\mathcal{G}) = \mathrm{mf}(\mathcal{M})$ is called the *minimal filling*. It was proved in [2] that the change of a metric ρ by the metric $\lambda\rho + a, \lambda > 0, a > \lambda a_\rho$, where a_ρ is a number dependent on the metric ρ, does not change the type of minimal filling.

The general problem was formulated as follows: given a class F of metric spaces and a family T of transformations of the metrics from F, describe the family T' of all transformations from T mapping F into itself and preserving some types of minimal fillings. We put $\rho_{ij} = \rho(p_i, p_j)$ and $\bar{\rho} = (\rho_{12}, \rho_{13}, \ldots, \rho_{n-1,n})$ for any metric space (M, ρ), where $M = \{p_1, \ldots, p_n\}$. Now we formulate our main results.

Theorem 1. *Let $f\colon \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ be a function such that for every metric space (M, ρ) the function $f \circ \rho$ is still a metric on M, and nondegenerate stars and types of minimal fillings of four-point spaces are preserved. Then there exists a real number C such that $f + 2C$ is linear on $\mathbb{R}_{>0}$.*

Theorem 2. *A matrix N of the form $A+B$, where A is a positive diagonal matrix and B is a matrix with identical rows of nonnegative elements, preserves metrics and minimal fillings whose types are nondegenerate stars, if and only if A is a scalar matrix.*

Theorem 3. *A linear mapping A takes additive metric spaces of at least four points to additive ones with the same non-degenerate type of minimal filling if and only if A has the form $\bar{\rho} \to \alpha\bar{\rho}$, $\alpha > 0$.*

Theorem 4. *A matrix of a one-to-one linear mapping that takes each ultrametric space of 3 points to an ultrametric space has the form $A = R(B + \lambda E)$, where B is a matrix of identical rows of positive elements, $\lambda \in \mathbb{R}$, and R is a permutation of the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.*

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Optimal position of compacts in the spaces with Euclidean Gromov-Hausdorff metric

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We study nonempty compact subsets of Euclidean space disposed optimally (the Hausdorff distance between them cannot be reduced). We show that if one of them is a singleton, then it coincides with the Chebyshev center of the second one. We also consider many other particular cases. As an application, we show that each three-point metric space can be isometrically embedded in the orbits space of the group of proper motions acting on the compact subsets of Euclidean space. In addition, we prove that for each couple of optimally located compacts, all compacts intermediate in the sense of Hausdorff metric, are intermediate in the sense of Euclidean Gromov–Hausdorff metric as well.

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String topological robotics 2

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We claim here to link two well known theories; namely the string topology (founded by M. Chas and D. Sullivan in 1999) and the topological robotics (founded by M. Farber some few years later, in 2003). For our propose purpose, we consider G a compact Lie group acting transitively on a path connected n- manifold M. On the set MLP(M) of the loop motion planning algorithms, they define a string loop motion planning product, that endows the shifted homology $H_?(MLP(M)) := H_{?+2n}(MLP(M))$ with a structure of a commutative and associative graded algbera. We finish the task and extend this structure to a Gerstenhaber algebra one and discuss how to do the same toward a Batalin-Vilkovisky algebra structure

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On manifolds with almost hypercomplex structures and almost contact 3-structures, equipped with metrics of Hermitian-Norden type

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The report provides an overview of the author's latest results on this topic.

In the beginning, some facts are given concerning the almost hypercomplex manifolds with Hermitian-Norden metrics known from the papers of the author, K. Gribachev and collaborators.

Next, integrable hypercomplex structures with Hermitian-Norden metrics on Lie groups of dimension 4 are considered. The corresponding five types of invariant hypercomplex structures with hyper- Hermitian metric are constructed here. The different cases regarding the signature of the basic pseudo-Riemannian metric are considered.

Further, the tangent bundle of an almost Norden manifold and the complete lift of the Norden metric are considered as a $4n$-dimensional manifold. It is equipped with an almost hypercomplex Hermitian-Norden structure and characterized geometrically. The case when the base manifold is an h-sphere is considered.

Then, it is introduced an associated Nijenhuis tensor of endomorphisms in the tangent bundle of an almost hypercomplex manifold with Hermitian-Norden metrics. There are studied relations between the six associated Ni-

jenhuis tensors of an almost hypercomplex structure as well as their vanishing. It is given a geometric interpretation of the associated Nijenhuis tensors for an almost hypercomplex structure and Hermitian- Norden metrics. Finally, an example of a 4-dimensional manifold of the considered type with vanishing associated Nijenhuis tensors is given.

Next, quaternionic Kähler manifolds corresponding to almost hypercomplex manifolds with Hermitian-Norden metrics are considered. Some necessary and sufficient conditions for the studied manifolds to be isotropic hyper-Kählerian and flat are found. It is proved that the quaternionic Kähler manifolds with the considered metric structure are Einstein for dimension at least 8. The class of the non-hyper-Kähler quaternionic Kähler manifold of the considered type is determined.

In the second part of this report, it is introduced a differentiable manifold with almost contact 3-structure which consists of an almost contact metric structure and two almost contact B-metric structures. The corresponding classifications are discussed. The product of this manifold and a real line is an almost hypercomplex manifold with Hermitian-Norden metrics. It is proved that the introduced manifold of cosymplectic type is flat. Some examples of the studied manifolds are given.

Finally, it is considered a differentiable manifold equipped with a pseudo-Riemannian metric and an almost contact 3-structure so that one almost contact metric structure and two almost contact B-metric structures are generated. There are introduced associated Nijenhuis tensors for the studied structures. The vanishing of the Nijenhuis tensors and their associated tensors is considered. It is given a geometric interpretation of the vanishing of associated Nijenhuis tensors for the studied structures as a necessary and sufficient condition for existence of affine connections with totally skew-symmetric torsions preserving the structure. An example of a 7-dimensional manifold with connections of the considered type is given.

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Recent results on the groups $\mathbf{G_n^k}$

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In 2015, the author defined a family of groups depending on two integer parameters, $n > k$, and formulated the following principle:

If dynamical systems describing a motion of n particles possess a good property governed by exactly k particles then they have topological invariants valued in G_n^k.

We shall discuss the recent progress on G_n^k from the following points of view:

1) Topology: how to study configuration spaces and manifolds by using these groups and their close relatives

2) Algebra: how to solve word, conjugacy problems in these groups and how they are related to braid groups, Coxeter groups and other famous groups.

3) Geometry and dynamics: how to modify the above principle to make it more sensitive to the phase space/geometrical structures on manifolds

as well as many other aspects of these groups

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Geometrical analysis of a unit interval experiment

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In this paper, we find an algorithm for determination of a function which depends on a given nonnegative sequence and a variable t. This variable is used as upper bound for the linear combination of elements of known sequence and the elements of a random sequence from the unit interval. The value of the function is expected value of the number of the successive elements of the random sequence taken such that the linear combination exceed t. After geometrical evident studied cases, we find the function in the series form. Finally, for a few particular sequences, we recognize a few known functions.

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Schwarz lemma and Kobayashi metrics for harmonic and holomorphic functions

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In this note we mainly consider various version of Schwarz lemma and its relatives related to harmonic and holomorphic functions including several variables. It turns out that our methods (results) unify very recent approaches. In particular our considerations include domains on which we can compute Kobayashi-Finsler norm.

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Semi-slant submanifolds in a locally conformal Kaehler space form

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In 1994, N. Papaghiuc introduced the notion of a semi-slant submanifold in a Hermitian manifold which is a generalization of CR and slant submanifolds. In this talk, we we consider semi-slant submanifolds in a locally conformal Kaehler space form. We mainly give inequalities of the length of the second fundamental form and the mean curvature. Then, using the Codazzi equation, we show the tensor field P define in (1.4) in a semi-slant submanifold with parallel second fundamental form satisfies a special equation with respect to a generalized adapted frame.

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On geodesic mappings and their generalizations

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In the lecture will be about new results in the theory of geodesic mappings and their generalizations.

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About almost geodesic curves

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By an *almost geodesic* of an affine connection ∇ we mean a piecewise C^3-curve $\gamma\colon I \to \mathbb{R}^n$ satisfying $\nabla_{\dot\gamma}(\nabla_{\dot\gamma}\dot\gamma) = \varrho\cdot\dot\gamma + \sigma\cdot\nabla_{\dot\gamma}\dot\gamma$, where ϱ, $\sigma\colon I \to \mathbb{R}$ are continuous functions, $I \subset \mathbb{R}$ is an open interval.

We consider a curve $\mathcal{C}$ homeomorphic to $\mathbb{R}$ which is a closed subset of $\mathbb{R}^n$ and has the form $\mathcal{C} = (t, f_2(t), \dots, f_n(t)), t \in \mathbb{R}$, where $f_i(t)\colon \mathbb{R} \to \mathbb{R}$, $i = 2, \dots, n$, are three times differentiable non-constant functions. The system $\mathfrak{X}(\mathcal{C}) = \{(t + c_1, b_2 f_2(t) + c_2, \dots, b_n f_n(t) + c_n), t \in \mathbb{R}\}$, where $b_i \neq 0, c_i \in \mathbb{R}$, is a set of imagines of $\mathcal{C}$.

Let $\ell = (t + c_1, b_2\, f_2(t) + c_2, \dots, b_n\, f_n(t) + c_n),\ t \in \mathbb{R}$, be a curve of $\mathfrak{X}(\mathcal{C})$. A curve ℓ of $\mathfrak{X}(\mathcal{C})$ is an almost geodesic with respect to a connection ∇ with constant coefficients $\{\Gamma^h_{ij}\}$ if and only if we have

$$\dddot{\ell}^h + \sum_{i,j,k=1}^{n} \Gamma^\ell_{ij}\Gamma^h_{\ell k}\,\dot\ell^i\dot\ell^j\dot\ell^k + 2\sum_{i,j=1}^{n}\Gamma^h_{ij}\ddot\ell^i\dot\ell^j + \sum_{i,j=1}^{n}\Gamma^h_{ij}\dot\ell^i\ddot\ell^j =$$

$$\varrho(t)\cdot\dot\ell^h + \sigma(t)\cdot(\ddot\ell^h + \sum_{i,j=1}^{n}\Gamma^h_{ij}\dot\ell^i\dot\ell^j).$$

We obtain the form of curves $\mathcal{C}$ in the n-dimensional real space $\mathbb{R}^n$ which are almost geodesics with respect to an affine connection ∇ and calculate explicitly the components of ∇. In doing so we supposed that with $\mathcal{C}$ also all images of $\mathcal{C}$ under a real $(n-1)$-dimensional algebraic torus are also almost

geodesics. This implies that the determination of $\mathcal{C}$ becomes an algebraic problem.

If every curve of $\mathfrak{X}(\mathcal{C})$ is an almost geodesic with respect to ∇, then the derivatives $f_i'(t)$ are solutions of harmonic oscillator equations. If $\mathfrak{X}(\mathcal{C})$ consists of Euclidean lines which are geodesic with respect to ∇, then at the most Γ^1_{11} may be different from 0. In contrast to this if $\mathfrak{X}(\mathcal{C})$ consists of Euclidean lines then there is huge quantity of non-trivial connections ∇ such that the lines of $\mathfrak{X}(\mathcal{C})$ are almost geodesic with respect to ∇.

Here we consider the case, when $\Gamma^h_{11} = \Gamma^1_{11}$ for all $2 \leq h \leq n$ and $\Gamma^h_{i1} + \Gamma^h_{1i} = \Gamma^1_{i1} + \Gamma^1_{1i}$ for all $2 \leq i \leq n$, but there exists an α and i_0, j_0 such that $\Gamma^\alpha_{i_0 j_0} \neq \Gamma^1_{i_0 j_0}$.

This work is a continuation of the researches of professor Karl Strambach who suddenly died in October, 2016.

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On hyperspace mappings

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Let X, Y be nonempty compact subspaces of metric space Z then the value $|XY|_Z = \max\{\sup\limits_{x\in X}\inf\limits_{y\in Y}|xy|, \sup\limits_{y\in Y}\inf\limits_{x\in X}|yx|\}$ is called the Hausdorff distance between X and Y.

Assume that X and Y are metric spaces. A triple (X', Y', Z) consisting of metric space Z and two its subspaces X' and Y' isometric to X and Y, respectively, is called a realization of the pair (X, Y). The Gromov–Hausdorff distance $d_{GH}(X, Y)$ between X and Y is the infinum of real numbers r, for which there exists a realization (X', Y', Z) of (X, Y) with $|X'Y'|_Z \leq r$.

The set $\mathcal{M}$ of all isometry classes of compact metric spaces with Gromov-Hausdorff metric is called the Gromov-Hausdorff space.

By $CL(X)$ we denote the set of all nonempty and closed subspaces of topological space X. Any subset of $CL(X)$ is called a hyperspace. By $F_n(X)$ we denote the set of all subspaces of topological space X with cardinality at most n and by $\mathcal{H}(X)$ — the family of all compact subspaces.

Let X be a metric space then the sets $\mathcal{H}(X)$ and $F_n(X)$ for any natural n endowed with Hausdorff distance are metric spaces and are compact if X is compact.

Any mapping $f : \mathcal{M} \to \mathcal{M}$ taking each metric compact to some of its hyperspace is called hyperspace mapping. The mapping $X \mapsto \mathcal{H}(X)$ is called the Hausdorff mapping and the mapping $F_n : X \mapsto F_n(X)$ is called n-fold symmetric product mapping.

The general goal is to study the properties of hyperspace mappings. In the talk it is shown that the mappings $\mathcal{H}$ and F_n for any n are Lipschitz with constant 1 and hence continuous. Also it will be shown which metric properties and characteristics are preserved by mappings $\mathcal{H}$ or F_n and which are not.

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Mathematical Conference: XX Geometrical Seminar
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The growth of polynomial Lie algebras

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There is an algebraic construction which generalises the properties of the Lie algebra $Vect^{\infty}(M)$ of vector fields on a smooth manifold M viewed as a module over the ring $C^{\infty}(M)$ of smooth functions on M. This general algebraic construction is known in literature as Lie-Reinhart algebra [1].

Let R be a commutative unital ring and A a commutative R-algebra. A pair $(A, \mathcal{L})$ is called a Lie-Reinhart algebra if

1) $\mathcal{L}$ is a Lie algebra over R which acts on (the left of) A (by derivations), i.e.

$$X(ab) = X(a)b + aX(b), \forall a, b \in A, \forall X \in \mathcal{L};$$

2) $\mathfrak{g}$ is an A-module.

The pair $(A, \mathcal{L})$ must satisfy the compatibility conditions that are

$$\begin{aligned} [X, aY] = X(a)Y + a[X, Y], \forall X, Y \in \mathcal{L}, \forall a \in A; \\ (aX)(b) = a(X(b)),\ \forall a, b \in A, \forall X \in \mathcal{L}. \end{aligned} \tag{1}$$

Buchstaber proposed in [2,3] to study a very important special case of graded Lie-Reinhart algebras $(A, \mathcal{L})$ when $A = R[t_1, t_2, \ldots, t_p]$ is a graded plolynomial algebra over R such that

1) $\mathcal{L}$ is a free left module of rang N over $R[t_1, t_2, \ldots, t_p]$.

2) $\mathcal{L} = \oplus_{i \in \mathbb{Z}} \mathcal{L}_i$ is a $\mathbb{Z}$-graded Lie algebra $[\mathcal{L}_i, \mathcal{L}_j] \subset \mathcal{L}_{i+j},\ i, j \in \mathbb{Z}$, and its grading is compatible with the grading of the algebra $R[t_1, t_2, \ldots, t_p]$.

$$p(t)L \in \mathcal{L}_{i+deg(p(t))},\ deg(L(q(t)) = deg(q(t)) + i, L \in \mathcal{L}_i.$$

where $p(t), q(t)$ are homogeneous polynomials in $R[t_1, t_2, \ldots, t_p]$ of gradings $deg(p(t))$ and $deg(q(t))$ respectively. Graded structure in $R[t_1, t_2, \ldots, t_p]$ is defined on generators

$$deg(t_1) = m_1, \ldots, deg(t_p) = m_p, m_i \in \mathbb{Z}.$$

The rang of the free left module $\mathcal{L}$ over the polynomial algebra $R[t_1, t_2, \ldots, t_p]$ is called the rang of the p-polynomial Lie algebra $(R[t_1, t_2, \ldots, t_p], \mathcal{L})$. We plan to discuss the growth of Lie subalgebras over R generated by the basis of the left free module $\mathcal{L}$ over the algebra $R[t_1, t_2, \ldots, t_p]$. The rate of growth is related to the integrabilty of some hyperbolic systems of PDE [3,4].

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Surfaces with parallel normalized mean curvature vector field in 4-spaces

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A basic class of surfaces in Riemannian and pseudo-Riemannian geometry are the surfaces with parallel mean curvature vector field, since they are critical points of some natural functionals and play important role in differential geometry, the theory of harmonic maps, as well as in physics. A natural extension of the class of surfaces with parallel mean curvature vector field are surfaces with parallel normalized mean curvature vector field.

On any surface with parallel normalized mean curvature vector field in a pseudo-Euclidean 4-space we introduce special geometric (canonical) parameters that allow us to describe this class of surfaces in terms of three invariant functions satisfying a system of three partial differential equations.

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On the existence of local quaternionic contact geometries

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In the talk I will present some recent results obtained in a collaboration with professor Jan Slovak from the Masaryk University (Czech Republic). In our work we exploit the Cartan-Kähler theory to prove local existence of real analytic quaternionic contact structures for any prescribed values of the respective curvature functions and their covariant derivatives at a given point on a manifold. We show that, in a certain sense, the different real analytic quaternionic contact geometries in 4n+3 dimensions depend, modulo diffeomorphisms, on 2n+2 real analytic functions of 2n+3 variables.

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On one problem in a space of non-symmetric affine connection and its subspace

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Let X_N be a submanifold of a differentiable manifold $X_N(X_M \subset X_N)$. If we have defined non-symmetric affine connection L on X_N, given by coefficients $L^i_{jk} \neq L^i_{kj}$, and on X_M non-symmetric basic tensor g in the paper we consider the question (stated by M. Prvanović in personal communication): Find a connection between induced connection $\bar{L}$ from L_N in L_M $(L_N = (X_N, L))$ and the connection $\bar{\Gamma}$, defined by g in X_M. The solution is given and some examples are constructed.

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Smooth version of Johnson's problem concerning derivations of group algebras

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A description of the algebra of outer derivations of a group algebra of a finitely presented discrete group is given in terms of the Cayley complex of the groupoid of the adjoint action of the group. This task is a smooth version of Johnson's problem concerning the derivations of a group algebra. It is shown that the algebra of outer derivations is isomorphic to the group of the one-dimensional cohomology with compact supports of the Cayley complex over the field of complex numbers.

The report presents the results obtained jointly with A. Arutyunov, and also with the help of A.I. Shtern

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On compact non-Euclidean polyhedral manifolds - theory, applications and visualization

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Abstract. As a by-product of our recent papers [2], [3], [4], and the previous initiative of the first author [1], we have recently found an infinite sequence of hyperbolic polyhedra Cw(2z, 2z, 2z) ($6 \leq 2z$, $3 \leq z$ odd integer) each of them can be equipped with a fixed point free face pairing, as a gluing procedure, so that the tube-like polyhedron become a compact hyperbolic manifold. That means each point has a ball-like neighbourhood. The material applications seem to be timely: nanotubes, as new hyperbolic crystal structure in our experience space. Visualization of such "finite worlds" are also actual task. First, for introduction we consider the paper model of the famous hyperbolic football manifold, the Archimedean solid 10, 6, 4 (fullerene) and restrict ourselves only for Cw(6, 6, 6) and Cw(10, 10, 10) manifoldsabove, as in [4] and [5]. The algorithmic description of fundamental groups and other properties seem to be interesting problems, as well [5].A Nil packing and manifold [7], two hyperbolic space forms to the Archimedean solid 4, 6, 8 from [1]; and a brand new publication (but from 1990), a hyperbolic manifold (with three generators) to the Archimedean solid 10, 6, 4 will also be presented. Other geometries and algorithms will be reported as well [2], [6].
Key words: Fixed point free isometry groups of homogeneous spaces, infinite series of compact hyperbolic manifolds and possible material structures.
MSC 2010: 51F15, 52B15, 57S30

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Analytic exploration of rational and irrational numbers using geometry

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Rational numbers are a well explored area in the field of numbers and mathematics. But irrational still remain quite eluded. Could there be a geometric or analytic way to explore the topic deeper?

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Infinitesimal bending influence on the energies of knots

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This talk is devoted to a study of infinitesimal bending of the first and the second order of knotted curves. Variations of important geometric magnitudes describing the knots (the Willmore energy, the Möbius energy, the total curvature, the total torsion, the total normalcy) are exemined. Our visualization tool devoted to visual representation of infinitesimal bending and energy changes is presented.

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Bishop frame of a null Cartan curve in Minkowski space-time

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This contribution is a joint work with prof. Kazim Ilarslan (Kirikkale University, Turkey).

We define the Bishop frame, or relatively parallel adapted frame, of a null Cartan curve in Minkowski space- time $\mathbb{E}_1^4$ and obtain the Bishopś frame equations of a null Cartan curve which lies in the timelike hyperplane of $\mathbb{E}_1^4$. We show that a null Cartan cubic lying in the timelike hyperplane of $\mathbb{E}_1^4$ has two Bishop frames, one of which coincides with its Cartan frame. The Bishopś frame equations of the null Cartan curve which has the third Cartan curvature $\kappa_3(s) \neq 0$ are also derived. As an application, we find the solution of a null Betchov-Da Rios vortex filament equation in terms of a null Cartan curve and its Bishop frame.

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Non-compact singularities of integrable Hamiltonian systems with 2 degrees of freedom

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Consider a Liouville integrable Hamiltonian system with 2 degrees of freedom restricted to a regular isoenergy manifold Q^3 with a Bott-type additional integral. For compact manifolds Q^3, it was shown by A. T. Fomenko in [1] that a small invariant neighborhood of a singular leaf of the corresponding Liouville foliation (*3-atom*) possesses the structure of a Seifert fibration with singular fibers of the type $(2, 1)$. The base of this fibration can be realized as a small neighborhood of a singular level line of a Morse function on a compact 2-dimensional surface (*2- atom*). We prove that under some conditions this result is also true for the case of a non-compact manifold Q^3.

Theorem. Let L be a bifurcational leaf of the Liouville foliation corresponding to a Liouville integrable Hamiltonian system restricted to a non-compact regular isoenergy manifold Q^3. Let H be its Hamiltonian function and F its additional Bott-type first integral. Suppose that the Hamiltonian vector fields with the Hamiltonian functions H and F are complete, so that there is a well-defined Poisson action of the group $\mathbb{R}^2$ on Q^3 (by shifts along their integral trajectories) and the leaf L consists of 1-dimensional and 2-dimensional orbits of this action. Suppose further that one of these orbits is diffeomorphic to a circle or a cylinder $S^1 \times \mathbb{R}^1$. Then a small invariant neighborhood of the leaf L has the structure of a Seifert fibration with singular fibers of the type $(2, 1)$. The base of this fibration is a 2-atom, i.e. it can be realized as a small neighborhood of a bifurcational level line of a Morse function on a non- compact 2-dimensional surface.

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CUDA Realization for GC contour recognition method

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We compare speed and quality of CUDA realization for new contour detection algorithm based on geometrical coding (GC) with CUDA and OpenCL implementations (used in OpenCV library) of Canny operator which is commonly respected as the best modern algorithm for contour detection. The comparison shows that GC approach can really compete with Canny operator and in some cases even overcomes it in speed and in quality. Examples of GC contour detection in different situations are presented.

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Discrete geometric flows on two-dimensional compact oriented surfaces

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Geometric flows, or differential equations for families of time-dependent metrics, have been investigated in mathematics for a very long time. The Ricci flow and the mean curvature flow are the most commonly used examples [1], [2]. Perelman's proof of Thurston's conjecture on the geometrization of three-dimensional manifolds and, in particular, the Poincare conjecture, is considered to be the most effective application of Ricci flows [3]. Theorems on the convergence of the Ricci flow on two-dimensional closed oriented surfaces to the metric of constant curvature were among the first to be proved. Initially, Hamilton had proved a theorem on the convergence of the Ricci flow for any initial metric to a metric of constant curvature on a two-dimensional compact oriented and not diffeomorphic to a sphere surface, while a similar theorem has been proved for a surface diffeomorphic to a sphere under the condition of the positive Gaussian curvature of the surface [4]. Later on, a theorem on the convergence of the Ricci flow to the metric of constant curvature for any metric on the sphere was proved by Chow [5]. Currently several approaches to the discretization of Ricci flows exist [6]. This work discusses the most natural way of discretization by the Ricci flows on two-dimensional surfaces proposed by Chow and Luo. In their approach the Thurston's circle packing metric [6], [7] is considered as a metric on triangulated surface. In [8] it has been demonstrated that the Ricci flow on surfaces in the combinatorial setting converges to the Thruston circle packing metric. The combinatorial structure of a triangulated surface also includes a set of weights on all its edges. A crucial assumption for the convergence of the

flow in [8] was the condition of non-negativity of these weights. This paper demonstrates that the theorem of convergence of the Ricci flow to a unique metric of constant curvature can be proved under weakened conditions on the weights. Examples of weight sets on the edges of a triangulated surface allowing the existence of several metrics of constant curvature, are also demonstrated. Moreover, a numerical simulation of the flow in the vicinity of these solutions is presented. The discretization of the mean curvature flow on the surface of revolution, given by a special partition of the icosahedron[9], is also demonstrated. The numerical solution demonstrates the formation of singularities by the mean curvature flow for the case when the surface does not satisfy the conditions of Theorem 1.1 [2].

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The QC Yamabe problem on non-spherical quaternionic contact manifolds

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The well–known Yamabe problem in Riemannian geometry concerns the existence of a metric of constant scalar curvature of the conformal class of Riemannian metrics on a manifold [2]. In this talk we consider a sub-Riemannian version of this problem in a quaternionic contact (qc) setting. Namely, we demonstrate that the qc Yamabe problem has a solution on any compact qc manifold which is non-locally qc equivalent to the standard 3-Sasakian sphere. Precisely, we establish that the qc Yamabe constant of any such manifold is strictly less than the corresponding constant of the 3-Sasakian sphere, which allows us to give an affirmative answer to the qc Yamabe conjecture [1].

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Computational geometry as a tool for studying root-finding methods

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We present an efficient method come from Computational geometry, a branch of computer science devoted to the study of algorithms, for mathematical visualization of a third order root solver. For many decades the quality of iterative methods for solving nonlinear equations were analyzed only by using numerical experiments. The disadvantage of this approach is the inconvenient fact that convergence behavior strictly depends on the choice of initial approximations and the structure of functions whose zeros are sought, which often makes the convergence analysis very hard and incomplete. For this reason in this paper we apply dynamic study of iterative processes relied on basins of attraction, a new and powerful methodology developed at the beginning of the 21th century. This approach provides graphic visualization into the behavior of convergent sequences and, consequently, offers considerably better insight into the quality of applied root solvers, especially into the domain of convergence. For demonstration, we present dynamic study of one parameter family of Halleyś type introduced in the first part of the paper. Characteristics of this family are discussed by basins of attractions for various values of the involved parameter. Special attention is devoted to clusters of polynomial roots, one of the most difficult problems in the topic. The performed analysis and drawing basins of attractions are performed by the computer algebra system *Mathematica*.

AMS Mathematical Subject Classification (2010): 65H05, 65D18, 68W30, 33F05
Key words and phrases: Computational geometry; Parametric iterative methods; Graphic visualization; Dynamic study; Basin of attraction.

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Equitorsion holomorphically projective mappings of generalized m-parabolic Kähler manifolds

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We investigate equitorsion holomorphically projective mappings of generalized m-parabolic Kähler manifolds and provide some necessary and sufficient conditions for the existence of these mappings in form of linear PDE-systems.

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Novel method for computing approximative distance from a voxel to a twice differentiable surface

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In our previous work on breast and material simulation, we extensively utilized an approximative distance from a voxel to a quadratic surface based on the minimal and maximal distance of a vertex to the surface. It was observed that this approximation can lead to artifacts visible in simulated 3D objects. In this work, we develop theoretical framework that explains this observed behavior and propose a novel approximation, based on utilization of the circumscribed sphere around the voxel. We generalize the approach to computation of minimal and maximal distance to a twice differentiable surface and demonstrate the effects of utilizing the new method in breast simulation.

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Algebraic K-theory of generalized triangular matrix rings

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Assume A is an associative ring with 1. Quillen defined higher algebraic K–groups for the ring A as the homotopy groups of the 'plus-construction' to $BGL(A)$:

$$K_i(A) = \pi_i(BGL^+(A)), i \geq 1.$$

Remind that also one has $K_0(A)$ which is defined in terms of finitely generated projective left modules over A.

Consider the ring A_2 of 2-by-2 upper triangular matrices over A. In 1974 Quilllen in his lectures in Oberwolfach announced natural isomorphims $K_i(A_2) = K_i(A) \oplus K_i(A)$. Then Dennis and Geller [1] generalized this statement as follows. Let A and B are two associative rings with 1. Assume M is a A-left and B-right bimodule. Then one can form a new ring R of matrices $\begin{pmatrix} a & m \\ 0 & b \end{pmatrix}$ where $a \in A$, $b \in B$, $m \in M$. Dennis and Geller proved that $K_i(R) = K_i(A) \oplus K_i(B)$ for $i = 0, 1, 2$. In [2] Berrick and Keating proved that there is a natural isomorphims $K_i(R) = K_i(A) \oplus K_i(B)$ for all $i \geq 0$. Their original proof was based on investigation of homology groups of $BGL(R)$. In [3] Keating published much shorter proof based on the another (but equivalent) definition of the higher algebric K-groups and the calculus of functors on the category of finitely generated projective left R-modules.

These results by obvious induction argument are true for the ring of upper triangular n-by-n matrices.

In our talk we discuss similar statement for much more general rings which are formed in a tensor-like way. We define upper triangular tensor-like rings and prove that K_i-group of such a ring for all i is the direct sum of K_i-groups of diagonal part of the ring.

This results appeared to be importnant for the calculations of the higher polythopal K-groups.

The author was supported by the Russian Foundation for Basic Research (grant No. 16-01-00378-a) and the program "Leading Scientific Schools" (grant no. NSh-6399.2018.1).

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Critical metrics of the volume functional on compact manifolds with boundary

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In this talk we discuss the space of smooth Riemannian structures on compact manifolds with boundary that satisfies a critical point equation associated with a boundary value problem, for simplicity, V-static metrics (cf. [1,2]). It is known that V-static metrics are important in understanding the interplay between volume and scalar curvature. We provide an estimate to the area of the boundary of V-static metrics on compact three-manifolds. Moreover, we present a Bochner type formula which enables us to classify critical metrics of the volume functional on a compact manifold with boundary. In addition, we show that Bach-flat critical metric of the volume functional on a compact manifold with boundary must be isometric to a geodesic ball in a space form.

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Variations of the Godbillon-Vey invariant of foliated 3-manifolds

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Variations of the Godbillon-Vey invariant of foliated 3-manifolds Abstract: Let M be a smooth three-dimensional manifold equipped with a vector field T transverse to a plane field $\mathcal{D}$ – the kernel of one form ω such that $\omega(T) = 1$. In [Rovenski V. and Walczak P. A Godbillon–Vey type invariant for a 3-dimensional manifold with a plane field, ArXiv:1707.04847], we constructed a three- form analogous to that defining the Godbillon-Vey class of a foliation, showed how does this form depend on ω and T, and deduced Euler-Lagrange equations of the associated functional. In this talk, we consider singular distibutions/foliations and forms that is those defined outside a finite union of pairwise disjoint closed submanifolds of codimension greater than 2 under additional assumption of convergence of certain integrals. We find derivatives of the functional and characterize critical foliations for different types of variations, prove sufficient conditions for critical pairs when $\mathcal{D}$ varies over foliations, and provide some examples with Reeb foliations and twisted products.

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Rotary mappings onto spaces with affine connection

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We obtain new results of rotary mappings of two-dimension (pseudo-) Riemannian onto spaces with affine connection.

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Infinitesimal rotational transformations

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The paper is devoted to further study of a certain type of infinitesimal transformations of (pseudo-) Riemannian spaces, which are called rotational. An infinitesimal transformation is called rotational if it maps geodesics in (pseudo-) Riemannian space onto isoperimetric extremals of rotation in their principal parts in (pseudo-) Riemannian space. The basic equations of such transformations are deduced and studied in detail.

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Recurrent equiaffine projective Euclidean spaces

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In the work by P. A. Shirokov there have been studied symmetric and semisymmetric projective Euclidean spaces.

Our work is devoted to recurrent projective Euclidean spaces.

Recurrent spaces are characterized by absolute recurrence of the curvature tensor. If a function φ is gradien-like, then recurrent space is semi symmetric.

Semi symmetric spaces are spaces with affine connection in which curvature tensor R satisfies the following condition $R \circ R = 0$.

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The full symmetric Toda system on Lie algebras and Bruhat order

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The full symmetric Toda system is determined by the Cartan decomposition of a real Lie algebra. It can can be described with the help of gradient a dynamic system on the flag space of the corresponding Lie group. In my talk I will describe the invariant surfaces of this system, associated with the representations of the Lie algebra. In many situations these surfaces allows one describe the phase portrait of the system in the terms of Bruhat order of the corresponding (relative) Weyl group.

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Nonlocal modified Einstein gravity

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Although very successful, Einstein theory of gravity is not a final theory. There are many its modifications, which are motivated by quantum gravity, string theory, astrophysics and cosmology. One of very promising directions of research is *nonlocal modified gravity* and its applications to cosmology.

In this contribution we consider model of nonlocal gravity without matter, given by the following action

$$S = \frac{1}{16\pi G} \int \sqrt{R - 2\Lambda}\, F(\Box) \sqrt{R - 2\Lambda}\, \sqrt{-g}\, d^4x,$$

where R is Ricci scalar curvature, Λ cosmological constant and $F(\Box) = 1 + \mathcal{F}(\Box) = 1 + \sum_{n=1}^{\infty} f_n \Box^n$. The corresponding Einstein equations of motion are derived and presented. Investigation of equations of motion and finding its solutions is a very difficult task. Using ansatze we can simplify the problem and get some cosmological solutions.

This is joint work with I. Dimitrijevic, B. Dragovich and Z. Rakic.

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Certan properties of second type almost geodesic mappings of nonsymetric affine connection spaces

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We consider second type almost geodesic mapping of the spaces with a nonsymmetric affine connection. Special attention has been paid to almost geodesic mappings with the property of reciprocity and also to the canonical almost geodesic mappings. Some invariant geometric objects were found for these mappings.

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Generalized Einstein spaces in the sense of Eisenhart

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In the present paper are obteined some relations of Einstein type tensors of the first and the second kind. Generalized Einstein spaces of the kind $\theta\,(\theta = 1, 2)$ in the sense of Eisenhart are introduced. Eisntein type tensors are represented in the generalized Einstein spaces. Also, geodesic mappings of T-connected generalized Einstein spaces onto Riemannian space are considered.

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The Sampson Laplasian acting on covariant symmetric tensors

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Forty five years ago J. H. Sampson has defined a Laplacian operator acting on covariant symmetric tensors [1]. This operator was an analogue of the well known Hodge- de Rham Laplacian which acts on exterior differential forms. We study the Sampson Laplacian using the analytical method, due Bochner, of proving vanishing theorems for the null space of a placeLaplace operator admitting a Weitzenböck decomposition and further of estimating its lowest eigenvalue.

Theorems and corollaries of the report complement our results from [2]; [3]; [4] and [5].

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Bifurcations of Steiner minimal networks and minimal fillings

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This talk will be about Steiner minimal trees and minimal fillings for finite sets of points in metric spaces. A Steiner minimal tree is the shortest tree connecting a finite set of points in a metric space with all vertices are in the space. A minimal filling is a connecting weighted graph of the minimal weight where non-negative weight function has some restrictions caused by metric.

Constructing of these objects are NP-hard problems in many metric spaces, including the Euclidean plane. But their solutions help us to design transport networks, build phylogenetic trees, etc.

It is important to say about topologies of these two objects for finite sets of points in a metric space. In this work bifurcation diagrams of Steiner minimal trees and minimal fillings for any four points in the Euclidean plane are construsted, and some restrictions on such diagrams for arbitrary sets are imposed.

The difference between the length of a Steiner minimal tree and the weight of minimal filling can be evaluated with the Steiner subratio. In this talk the Steiner subratios of smooth Riemannian manifolds will be estimated.

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Anisotropic image evolution of Synge-Beil type

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The anisotropic Beltrami framework is presented as a useful geometrical tool in image processing. Image surface evolution is provided by an anisotropic flow provided by a Polyakov energy Lagrangian. Particularly, the Synge-Beil flow is derived. Applicative aspects will be considered. This is joint work with Vladimir Balan.

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Hyperbolic space groups with truncated simplices as a fundamental domains

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In the papers of I.K. Zhuk, then more completely of E. Molnár, I. Prok, J. Szirmai all simplicial 3-tilings have been classified, where a symmetry group acts transitively on the simplex tiles. The involved spaces depends on some rotational order parameters. When a vertex of a such simplex lies out of the absolute, e.g. in hyperbolic space H^3, then truncation with its polar plane gives a truncated simplex or simply, trunc-simplex.

In previous papers of E. Molnár and M. Stojanović are given numerous casses of groups with trunc-simplical domains. In order to finish this classification here are investigated 5 new cases of such groups from different non-maximal families and 12 groups from maximal ones.

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Pseudo-symmetries of generalised Wintgen ideal Legendrian submanifolds

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For Legendrian submanifolds M^n in Sasakian space forms $\widetilde{M}^{2n+1}(c)$, I. Mihai obtained an inequality relating the main intrinsic and extrinsic scalar invariants, namely the normalised scalar curvature (intrinsic invariant) and the squared mean curvature and the normalised scalar normal curvature of M in the ambient space $\widetilde{M}$ (extrinsic invariants) which is called the *generalised Wintgen inequality*, characterising also the corresponding equality case. And a Legendrian submanifold M^n in Sasakian space forms $\widetilde{M}^{2n+1}(c)$ is said to be *generalised Wintgen ideal Legendrian submanifold* of $\widetilde{M}^{2n+1}(c)$ when it realises at everyone of its points the equality in such inequality. Characterisations based on some basic intrinsic symmetries involving the Riemann–Cristoffel curvature tensor, the Ricci tensor and the Weyl conformal curvature tensor belonging to the class of *pseudo-symmetries in the sense of Deszcz* of such *generalised Wintgen ideal Legendrian submanifolds* are given.

Joint work with Miroslava Petrović–Torgašev and Anica Pantić.

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Geodesically equivalent metrics on homogenous spaces

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Two metrics on a manifold are geodesically equivalent if sets of their unparameterized geodesics coincide. In this paper we show that if two left G-invariant metrics of arbitrary signature on homogenous space G/H are geodesically equivalent, they are affinely equivalent, i.e. they have the same Levi-Civita connection. We also prove that existence of non-proportional, geodesically equivalent, G-invariant metrics on homogenous space G/H implies that their holonomy algebra cannot be full. We give an algorithm for finding all left invariant metrics geodesically equivalent to a given left invariant metric on a Lie group. Using that algorithm we prove that no two left invariant metric, of any signature, on sphere S^3 are geodesically equivalent. However, we present examples of Lie groups that admit geodesically equivalent, non-proportional, left-invariant metrics.

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Path connectedness of spheres in Gromov-Hausdorff space

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We investigate path connectedness of spheres in Gromov- Hausdorff space. We prove the following two statements:

(1) Each sphere centered at one-point space is path connected;

(2) For any metric space X there exists a number R_X such that each sphere with center at X and radius greater than R_X is path connected.

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A couple of transverse $\mathbf{T^A}$-respecting foliations on Weil bundle $\mathbf{T^A}$ over a low-dimensional manifold and its construction

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Let M be an m-dimensional manifold, $x \in M$ be arbitrary and $A = \mathbb{R} \oplus N_A$ be a Weil algebra satisfying width $A = k \geq m$. We construct the so-called vertical foliation $\mathcal{V}$ on $\mathrm{reg}(T^r_k)_0\mathbb{R}^m$ and $\mathcal{V}^A$ on $\mathrm{reg}\, T^A_0\mathbb{R}^m$ and extend both of them to the natural systems of foliations $\mathcal{V}_{x,M}$ on $\mathrm{reg}(T^r_k)_xM$ and $\mathcal{V}^A_{x,M}$ on $\mathrm{reg}\, T^A_xM$.

For the jet group G^r_k and its subgroup $G^r_{k,m}$ of elements projectable to G^r_m we construct a global section $\hat{s} : G^r_{k,m}\backslash G^r_k \to G^r_k$ and the T^A-respecting map $s : G^r_{k,m}\backslash G^r_k \to \mathrm{reg}\, T^A_0\mathbb{R}^k$. For the subordinate map $s^\# : G^r_{k,m}\backslash G^r_k \to \mathrm{reg}\, T^A_0\mathbb{R}^m$ to s we construct the so-called horizontal foliation $\mathcal{H}^A_{s^\#}$ on $\mathrm{reg}\, T^A_0\mathbb{R}^m$, which is T^A-respecting. Any $\mathcal{H}^A_{s^\#}$ is extended to the natural system of T^A-respecting foliations $(\mathcal{H}^A_{s^\#})_{x,M}$ on $\mathrm{reg}\, T^A_xM$. It is proved that the leaves of vertical foliation $\mathcal{V}^A_{x,M}$ and the leaves of the horizontal foliation $(\mathcal{H}^A_{s^\#})_{x,M}$ intersect each other at exactly one element of $\mathrm{reg}\, T^A_xM$.

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Application of the geometry of curves in Euclidean space

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In this article using the geometry of curves in 3- dimensional Euclidean space, the induced spin velocities are studied, which were recently introduced by the author. Some essential properties of them are given, and they are rather different than the ordinary velocities. Indeed, the spin velocities are non-inertial, they are not constrained by the velocity of light, instead of the Lorentz transformations for them the Galilean transformations should be used, and in case of spin velocities the components of the electromagnetic tensor field remain unchanged. It is also given the method of calculation of the spin velocities, by using the curvatures and torsions of curves in 3-dimensional Euclidean space. Two important applications of the spin velocities are studied in many details.

MSC2010: 53A04, 51F25, 53Z05

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Symmetries in Gromov-Hausdorff Space

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We discuss the problem of finding local and global symmetries in the space of isometries classes of compact metrics spaces endowed with Gromov-Hausdorff distance.

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Simulation of any nondegenerate integrable system of general form with two degrees of freedom by the integrable topological billiard

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Let $X = (Q^3, v = sgradH)$ be an integrable Hamiltonian system with 2 degrees of freedom restricted to the compact and regular surface of constant energy $Q = \{H = h = const\}$. Defined *3-atom* as a small neighbourhood of a singular leaf of the Liouville foliation considered up to the fiber diffeomorphism.

Let A be the billiard, bounded by an arc of an ellipse, two arcs of confocal hyperbolas, and a focal straight line. This billiard is integrable. Denote the parameter the caustic by Λ. The value $\Lambda = b$ corresponds to the trajectories whose extensions pass through the foci. The result of gluing together several such billiards along convex arcs of boundaries ("spine") is called a billiard's book.

Theorem (I.Kharcheva, V.Vedyshkina) For any 3-atom, we can algorithmically construct the billiard's book, glued from elementary billiards of type A, such that the Liouville foliation on of the neighborhood of the singular value b for the integral Λ in the isoenergy surface Q^3 of this billiard is fiberwise homeomorphic to a given 3-atom.

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Second type almost geodesic mappings of special class and their invariants

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Invariants of almost geodesic mappings of a generalized Riemannian space are discussed in this paper. As a special case, invariants of equitorsion almost geodesic mappings of this type are discussed in here.

Key words: generalized Riemannian space, almost geodesic mapping, property of reciprocity, invariant, Thomas projective parameter, Weyl projective tensor.

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Manifolds associated with right-angled Coxeter groups

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Let R be the class of combinatorial 3-dimensional polytopes of simple combinatorial type, different from a tetrahedron, without 3- and 4-belts of faces. In particular, R contains the dodecahedron and fullerenes. By the results by Pogorelov (1967) and Andreev (1970), R is exactly the class of polytopes which can be realized in a hyperbolic 3-space with all dihedral angles to be right. Let G be Coxeter group, generated by reflection of faces of a polytope from R. We will discuss a method to construct hyperbolic 3-manifolds with fundamental groups commensurable with G. Various generalizations of the method and will be presented.

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Point cloud model of human bio form created by the application of geometric morphometrics: Human tibia example

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Morphometrics refers to the quantitative analysis of a biological form and it can be used to describe its shape. Common types of geometric morphometrics are Landmark-based Geometric Morphometrics which describe shape by using anatomical landmarks (e.g. points), and Outline-based geometric morphometrics which uses envelope curves to describe shape of the biological form (e.g. bone), and they are not absolutely exclusive. Geometric morphometrics can be used for the creation of statistical models which represent shape variation of specific bio form. In this paper, novel application of geometric morphometrics for the creation of personalized models of unique bio forms, i.e. models which are created for the specific patient is presented. Personalized model is defined as point cloud model of biological form (in this case human tibia). Model is formed by using interpolated envelope splines with landmark points defined by geometric morphometrics. Positions of points in 3D space are defined by using set of parametric functions defined

by applying geometrical morphometrics, morphology properties and statistical analysis on the input set of human tibia samples. By using this technique, anatomically correct and geometrically accurate personalized models of bio forms can be created and used in pre, intra, and post-operative procedures in clinical practice.

Keywords: geometric morphometrics, point cloud, statistical analysis, interpolated spline

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Jordan-Kronecker invariants for semidirect sums of semisimple Lie algebras with a commutative ideal

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Jordan–Kronecker invariants of a Lie algebra were first introduced by A. V. Bolsinov and P. Zhang in [1]. By definition, these invariants describe the canonical decomposition of a pair of skew-symmetric forms defined by the generic pair of elements of dual Lie algebra. For semisimple Lie algebras and Lie algebras of small dimension Jordan–Kronecker invariants are known; these results can be found in [1] and [3]. One interesting example of Lie algebras for which Jordan–Kroneker invariants are not yet calculated is semidirect sums of semisimple Lie algebras with several copies of space of standard representation.

It was proved by Bolsinov that the completeness of commutative family of shifts for a Lie algebra implies that this Lie algebra is of Kronecker type. Using this fact, the method was developed for calculating Jordan–Kronecker invariants for semidirect sums defined by standard representations of $so(n)$ and $sp(n)$.

This result is the subject of [2]. For semidirect sums defined by standard representations of $sl(n)$ or $gl(n)$, the complete answer is yet to be obtained, but in some special cases (for example, the case of $sl(n) \oplus (\mathbb{R}^n)^k$ with $k \geq n$) Jordan–Kronecker invariants were calculated with the help of the aforementioned method.

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On affinely equivalent left invariant metrics on Lie groups

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Two metrics of an arbitrary signature are called projectively equivalent if they have the same unparameterized geodesic lines. They are affinely equivalent if their Levi-Civita connections coincide. It is known that if two left invariant metrics on Lie groups are projectively equivalent they are also affinely equivalent. Classification of affinely equivalent metrics on Lie groups is still an open problem. As a step towards the classification, we construct nontrivial examples of such metrics.

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Lagrangian submanifolds of the complex hyperbolic quadric

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The complex hyperbolic quadric Q^{*n} is the complex hypersurface of complex $(n+1)$-dimensional hyperbolic space given in homogeneous coordinates by the equation $-z_0^2 - z_1^2 + \cdots + z_{n+1}^2 = 0$. This manifold inherits a Kähler structure from the complex hyperbolic space and carries an almost product structure. Its curvature can be relatively easily described in terms of these two structures.

Moreover, Q^{*n} is the natural target space when considering the Gauss map of a spacelike hypersurface of anti-de Sitter space $H_1^n(-1)$. In fact, such Gauss maps are related to minimal Lagrangian submanifolds of Q^{*n}.

Therefore we are particularly interested in the minimal Lagrangian immersions in the hyperbolic complex quadric. In particular, we classify all the minimal Lagrangians with constant sectional curvature.

The 1-dimensional quadric Q^{*1} is isometric to the 2-dimensional hyperbolic space H^2. The 2-dimensional quadric Q^{*2} is isometric to the Riemannian product of hyperbolic spaces $H^2 \times H^2$. Remark that such a relation doesnt́ exist for higher dimensions.

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Conform semi-invariant Riemannian maps from almost Hermitian manifolds

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Conformal semi-invariant Riemannian maps from almost Hermitian manifolds to Riemannian manifolds are investigated. We give examples, study the geometry of leaves of certain distributions. We also investigate certain conditions for such maps to be horizontally homothetic. Moreover, we introduce special pluriharmonic maps and obtain characterizations.

Keywords: Riemannian submersion, Conform Riemannian map.

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About slant geometry of spacelike submanifolds at least codimension two in de Sitter space

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By means of one-parameter families of Legendrian dualities in [3] which are the extensions of four Legendrian dualities defined in [2] for the pseudo-hyperspheres in Lorentz-Minkowski space, slant geometry of spacelike hypersurfaces in de Sitter space was studied in [1, 3]. It is clear that the results of [4] are the special cases of some of the results in [1, 3].

During this talk, taking into account the results in [5], slant geometry of spacelike submanifolds at least codimension two in de Sitter space will be constructed.

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Cohen-Macaulay modules over the algebra of planar quasi-invariants and Calogero-Moser systems

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My talk (based on a joint work with Igor Burban) is devoted to the algebraic analysis of planar rational Calogero-Moser systems. This class of quantum integrable systems is known to be superintegrable. This means that the underlying Schredinger operator with Calogero-Moser potential can be included into a large family of pairwise commuting partial differential operators such that the space of joint power series eigenfunctions is generically one-dimensional.

More algebraically, any such system is essentially determined by a certain algebro-geometric datum: the projective spectral surface (defined by the algebra of planar quasi-invariants with natural filtration) and the spectral sheaf (defined by a module known to be Cohen- Macaulay of rank one). This geometric datum has very special algebro-geometric properties, the most important of which is a very special form of the Hilbert polynomial of the module (sheaf). Moreover, the spectral variety appears to be rational but very singular (only Cohen-Macaulay, even not normal). It turns out that all rank one Cohen-Macaulay modules over the algebra of planar quasi-invariants can be explicitly described in terms of very natural moduli parameters, and this description looks in some sence very similar to to the description of the generalised Jacobian for singular rational curves. The spectral module of a planar Calogero-Moser system is actually projective, and its underlying moduli parameters are explicitely determined. Unlike the case of curves, not every Cohen-Macaulay module is spectral. The moduli

space of spectral sheaves appears to be much more subtle, but its structure indicates the existence of integrable deformations of Calogero-Moser systems. I am going to explain how the classification of CM modules, combined with tools of the algebraic inverse scattering method, leads to certain new integrable deformations of Calogero- Moser systems in the algebra of differential-difference operators.

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Liouville equivalence between system "Chaplygin ball with a rotor" and Zhukovsky case

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We consider the problem of a rolling balanced dynamically nonsymmetric ball with a rotor on a rough horizontal plane. Earlier A.Y. Moskvin constructed bifurcation diagrams of the momentum mapping and bifurcation complexes in order to study the dynamics of the system and find the singular solutions. A natural continuation of this research is the fine Liouville analysis of the system. We made one of the steps in this direction, namely, we found Fomenko invariants for this system and made rough topological analysis of the system.

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Modeling of minimal networks by means of linkages

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The main result of the work is the proof of existing 3−dimentional linkage that solves Steiner tree problem for any finite set of points on plain. This linkage shows us the topology of minimal network and positions of additional vertixes. If there are several minimal networks the linkage shows us all of them.

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Note on nonsymmetric affine connection space with auxiliary symmetric tensor

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The talk deals with the metric and non-metric connection. A space with a non-symmetric affine connection and a symmetric auxiliary tensor that is covariantly constant is analyzed. The conditions for the connection coefficients L^i_{jk} to generate metric space are given, ie that there is a basic tensor g_{ij} in L_N which is covariantly constant with respect to that connection.

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Nijenhuis tensor and almost geodesic mappings of the second type of Eisenhart spaces

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In the present paper are considered almost geodesic mappings of the second type of Eisenhart spaces. A new form of the basic equations of these mappings was found using the Nijenhuis tensor.

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