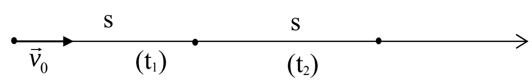
Први проблем

Крећући се праволинијски, тело прелази сукцесивно две деонице пута свака дужине S. Убрзање током кретања остаје исто, док време за које тело пређе прву деоницу је t_1, а другу $t_2>t_1$

a) Одредити почетну брзину v_0 и убрање a.
b) Колике су брзине на крају прве и друге деонице пута?
c) Колики је интервал времена до заустављања и пређени пут до заустављања, узети у обзир да је убрзање све време исто.
d) Нумеричке вредности: S=10m, t_1 =1,06 s, t_2=2,2s.

Решење првог проблема



a). Из $S = v_0 t_1 - \frac{a}{2}t_1^2$, $v_1 = v_0 - at_1$, $v_1^2 = v_0^2 - 2aS$, $S = v_1 t_2 - \frac{a}{2}t_2^2$,

добија се

$a = \frac{2S(t_2 - t_1)}{t_1 t_2 (t_2 + t_1)} = 2{,}99 m/s^2$, $v_0 = S\frac{t_2^2 - t_1^2 + 2t_1 t_2}{t_1 t_2 (t_2 + t_1)} = 11{,}02 m/s$. .... **2 поена**

b). $v_1 = S\frac{t_1^2 + t_2^2}{t_1 t_2 (t_1 + t_2)} = 7{,}84 m/s$, $v_2 = S\frac{t_1^2 - t_2^2 + 2t_1 t_2}{t_1 t_2 (t_1 + t_2)} = 1{,}25 m/s$. ....**2 поена**

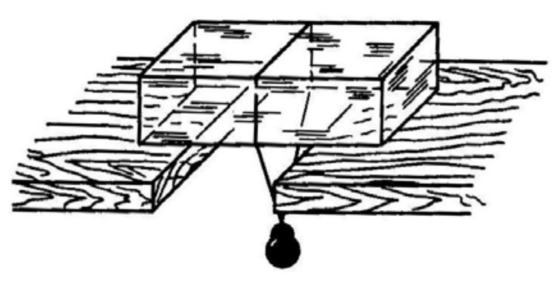
c). $\tau = t_1 + t_2 + t_{stop} = \frac{v_0}{a} = \frac{t_2^2 - t_1^2 + 2t_1 t_2}{2(t_2 - t_1)} = 3{,}68 s$ (т.j. $t_{stop} = \frac{v_2}{a} = 0{,}42 s$)

и $L = \frac{v_2^2}{2a} = S\frac{(t_1^2 - t_2^2 + 2t_1 t_2)^2}{4t_1 t_2 (t_2^2 - t_1^2)} = 0{,}26 m$ .... **2 поена**

d). За сваку нумеричку вредност 0,5 поена (помножено шестицом) ....**3 поена**
Екстра..**1 поена**
УКУПНО...**10 (десет) поена**

Problem 2

A). To determine the quality of thermal insulation of a Dewar vessel, it is filled with ice at
0^0C. After 24 hours, a quantity of 42 g of ice have melted. Usually liquid nitrogen at 78 K is kept in this flask. Assuming the quantity of heat entering the flask to be proportional to the difference in the internal and the external temperatures of the vessel, find the amount of liquid nitrogen that is going to evaporate in 24 hours. The ambient temperature is 20^0C. The heat of vapourization of liquid nitrogen at normal pressure is $L=1{,}8.10^5$J/kg.The specific heat of fuision for ice is $\lambda=3{,}4.10^5$ J/kg.



B). If you sling a thin wire loop around a block of ice and attach to it a weight of several kilograms then after some time the wire will pass through the block of ice, but the block remains intact (see figure). Explain this phenomenon. The ambient temperature is -2^{0}C and the atmospheric pressure is normal.

Solution

A). (6 points). The heat flowing to the Dewar vessel is $Q = \alpha(T_{air} - T)$, where α is a certain coefficient, and T is the temperature inside the flask**..1 point**

For ice and liquid nitrogen we obtain the ratio

$\frac{Q_1}{Q_2} = \frac{T_{air} - T_1}{T_{air} - T_2}$ **...1,5 points**

But, for nitrogen $Q_2 = m_2 L$, where L is its heat of evaporation and for ice $Q_1 = m_1\lambda$ **…………1 point**

Hence $\frac{m_1\lambda}{m_2 L} = \frac{T_{air} - T_1}{T_{air} - T_2}$, from which

$m_2 = \frac{m_1\lambda(T_{air} - T_2)}{L(T_{air} - T_1)}$ **...1,5 points**

Numerically $m_2 = 853$ grams**..1 point**

B).(3 points). Ice melts under the pressure of the wire, and the wire sinks; the water formed above the wire immediately freezes again.

Extra..1 point

Total...10 (ten) points.

Problem 3

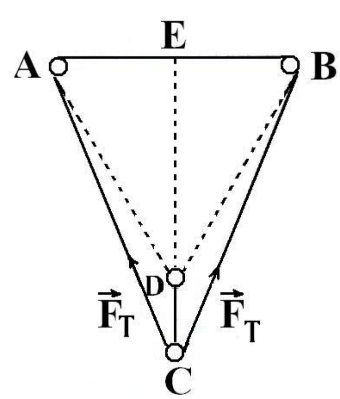
a).Ako je CD=x, AE=ℓ/2 i ED = $\ell\sqrt{3}/2$. Ukupno izduzenje je

$\Delta\ell = 2\sqrt{(\ell/2)^2 + \left(\ell\sqrt{3}/2 + x\right)^2} + \ell - 2\ell$ **...1 p**

b). Izraz za $\Delta\ell$ može da se napiše u obliku $\Delta\ell = \ell\sqrt{1 + \left(\sqrt{3} + 2x/\ell\right)^2} - \ell \approx \ell + x\sqrt{3}$ (primedba**:** za $x = 0$ se dobija da je $\Delta\ell = \ell$)........1 p

c). U stanju ravnoteže važi $mg = 2k\ell\cos(\pi/6)$ pa je $k = mg/2\ell\cos 30^0 = mg/\ell\sqrt{3}$ **..................................... 1punct**

d). Restitutciona sila je $F = 2F_T\cos\beta - mg$, gde je $\beta = ugao(ECB)$



$F_T = k\Delta\ell$, sa $\cos\beta = \dfrac{x + \frac{\ell}{2}\sqrt{3}}{\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{\ell}{2}\sqrt{3} + x\right)^2}}$ daje,

$$F = 2k\left(\ell\sqrt{1 + \left(\sqrt{3} + 2x/\ell\right)^2} - \ell\right)\frac{x + \frac{\ell\sqrt{3}}{2}}{\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(x + \frac{\ell\sqrt{3}}{2}\right)^2}} - mg$$ ....

. 2 p

e). Za mala pomeranja, se dobija $F_T = k\Delta\ell = \dfrac{mg}{\ell\sqrt{3}}\left(\ell + x\sqrt{3}\right)$,

$\cos\beta = \dfrac{x + \ell\sqrt{3}/2}{\sqrt{(\ell/2)^2 + \left(\ell\sqrt{3}/2 + x\right)^2}} \approx \dfrac{\sqrt{3} + 2x/\ell}{2}\dfrac{1}{\sqrt{1 + x\sqrt{3}/\ell}} \approx \dfrac{\sqrt{3}}{2} + \dfrac{x}{4\ell}$, tako da je, na kraju

$$F = 2\frac{mg}{\ell\sqrt{3}}\left(x\sqrt{3} + \ell\right)\left(\frac{\sqrt{3}}{2} + \frac{x}{4\ell}\right) - mg = 2\frac{mg}{\ell\sqrt{3}}\left(\frac{\ell\sqrt{3}}{2} + \frac{7x}{4}\right) - mg = \frac{7mgx}{2\ell\sqrt{3}}.$$ ....

2 p

f). Efektivna krutost žice je $k_{eff} = \dfrac{7mg}{2\ell\sqrt{3}}$ **.. 1 p**

g). Frekvencija malih oscilacija je $\omega = \sqrt{k_{eff}/m} = \sqrt{7g/2\ell\sqrt{3}}$ **..1 p**

Extra poen
.. 1 p

TOTAL...10 poena

Range 4th – 12th Class - Students
Solution to Problem No 4 (Version 3)

From equality relation between centripetal and Coulomb force acting on the electron in hydrogen atom

$$\frac{e^2}{4\pi\varepsilon_0 r_n^2} = \frac{mv_n^2}{r_n}, \qquad (1)$$

one can express velocity squared

$$v_n^2 = \frac{e^2}{4\pi\varepsilon_0 r_n m}. \qquad (2)\ \textbf{0.5 point}$$

From the relation for the angular momentum $L = m_e v_n r_n = n\frac{h}{2\pi}$ one can express radius of the n^{th} orbit r_n:

$$r_n = n\frac{h}{2\pi m_e v_n}. \qquad (3)$$

Combining eq. (3) with eq. (2) we get expressions for the electron's velocity and radius:

$$v_n = \frac{e^2}{4\pi\varepsilon_0 n\hbar}, \qquad (4)\ \textbf{0.5 point}$$

$$r_n = \frac{4\pi\varepsilon_0 n^2\hbar^2}{me^2}. \qquad (5)\ \textbf{0.5 point}$$

Total energy of the electron on the n^{th} orbit is then

$$E_n = \frac{mv_n^2}{2} - \frac{e^2}{4\pi\varepsilon_0 r_n} = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2 n^2}. \qquad (6)\ \textbf{0.5 point}$$

Photon was emitted during electron transition from k^{th} to n^{th} orbit ($k>n$). The energy of the photon is

$$E_\gamma = h\nu = E_k - E_n = \frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{k^2}\right) = hcR\left(\frac{1}{n^2} - \frac{1}{k^2}\right), \qquad (7)\ \textbf{1 point}$$

where R is Rydberg constant. In our case $k = 3$, $n = 1$:

$$E_\gamma = hcR\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1eV. \qquad (8)$$

Frequency of the photon is

$$\nu = \frac{E_\gamma.}{h} = 2.92\cdot 10^{15}\,Hz. \qquad (9)\ \textbf{0.5 point}$$

Using Einstein's celebrated relation for the photoelectric effect

$$E_\gamma = T + A_i, \qquad (10)$$

where $T = \frac{mv^2}{2}$ is kinetic energy of the photoelectron emitted from a surface of Lithium, one finds

$$T = E_\gamma - A_i = 9.81eV, \qquad (11)\ \textbf{1 point}$$

from which we obtain the velocity of emitted photoelectron

$$v = \sqrt{\frac{2T}{m}} = 1.857\cdot 10^6\,\frac{m}{s}. \qquad (12)\ \textbf{1 point}$$

De Broglie wave length of the photoelectron is obtained from De Broglie's relation

$$\lambda = \frac{h}{p}, \qquad (13)\ \textbf{0.5 point}$$

where $p = \sqrt{2mT}$ is non-relativistic momentum of the photoelectron. Finally

$$\lambda = \frac{h}{\sqrt{2mT}} = 392 \cdot 10^{-12} \, m \, . \qquad (14)$$ **1 point**

In the case of relativistic treatment

$$E_{TOT} = \frac{m_e c^2}{\sqrt{1 - \frac{v_{REL}^2}{c^2}}} = T + m_e c^2 \, , \qquad (15)$$ **0.5 point**

from which one obtains

$$v_{REL} = c \frac{\sqrt{T(T + 2mc^2)}}{T + mc^2} = 1.857 \cdot 10^6 \, \frac{m}{s} \qquad (16)$$ **1 point**

Finally

$$\frac{v}{v_{REL}} \approx 1 \qquad (17)$$ **0.5 point**

Extra **1 point**

TOTAL **10 points**