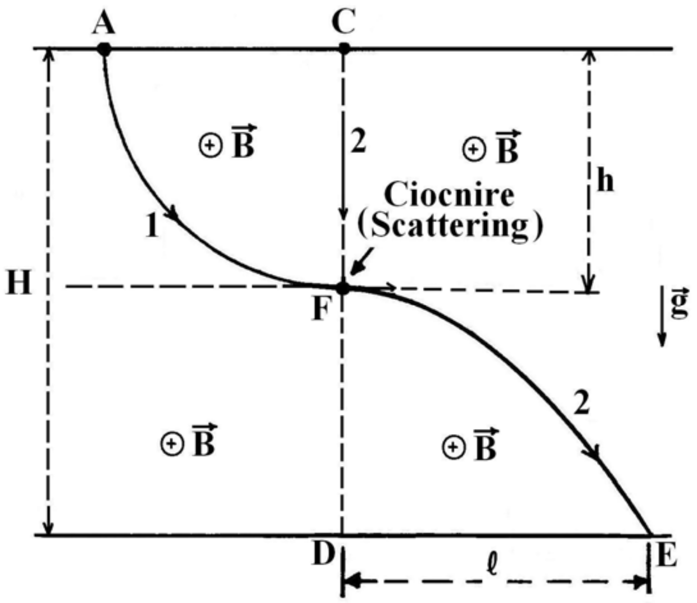
Third Class

First Problem

From the point A, situated at the height H (see the figure), a quasi-pointlike particle 1, of mass M and the electrical charge Q>0, is left free. The movement takes place into a homogeneous magnetic field, characterized by the magnetic induction $\vec{B}$, with the sense shown by the figure (towards the paper sheet, perpendicular to the drawing's plane) and in the gravity's field, characterized by the constant acceleration of gravity, $\vec{g}$. As it passes through the lowest point of its trajectory (point F in figure) the particle (1) scatters with a second quasi-pointlike particle (2), electrically not charged, i.e. neutral, which was left free from point C, situated on the same vertical line as the point F. The distance CF=h is known. Immediately after the scattering from the point F, the particle (1) moves on the horizontal direction, to the right, within the drawing plane. After the scattering, the particle (2) reaches to the point E, located through the distance DE=ℓ. Take into account the facts that no transfer of electrical charge has taken place, through the scattering, between the particles, and that the scattering was instantaneous, determine:



a). the time duration of the movement (motion) made by the particle (2) from point F to point E;
b). the mass of the particle (2).
c). the velocity of the particle (2) in the point E.

Solution for the first problem

a). Because the work of the Lorentz force $Q\left(\vec{v}\times\vec{B}\right)$ is always equal to zero, in the point F, before the scattering, we have $v_1 = v_2 = \sqrt{2gh}$ for both particles (with $\vec{v}_1$ to the right, in horizontal direction and $\vec{v}_2$ directed vertically to the down part)**...2 points**

We denote by $\vec{u}\left(u_x, u_y\right)$ the velocity of the particle (2) after the scattering. With the law of momentum conservation we can write $Mv_1 = Mv_1' + mu_x$ and also $mv_2 = mu_y$ (the horizontal axis (x) is directed to the right, the vertical axis (y)- to the down part). Then we find $v_2 = u_y$ and $m = M\frac{v_1 - v_1'}{u_x}$ **..2 points**

The total traveling time of the particle (2) can be expressed as $t_{CE} = t_{CF} + t_{FE}$, where $t_{FE} = \frac{\ell}{u_x}$, $t_{CE} = \sqrt{\frac{2H}{g}}$ and $t_{CF} = \sqrt{\frac{2h}{g}}$ .Therefore $u_x = \frac{\ell\sqrt{g}}{\sqrt{2H} - \sqrt{2h}}$. **,..1,5 points**

The answer to the first question (a) is $t_{FE} = \sqrt{\frac{2H}{g}} - \sqrt{\frac{2h}{g}}$ **..0,5 points**

b). In order to determine the velocity v_1' of the particle (1) ***immediately*** after the scattering we have in mind the information that $\vec{v}_1'$ (as vector) is directed horizontally to the right. This means that the Lorentz force QB v_1' (in "up" direction) equals the gravity force Mg (in "down" direction). In this way we find $v_1' = \frac{Mg}{QB}$ ....**1 point**

Now, with u_x and v_1' (in the relation obtained as a consequence of momentum conservation law) we get

$$m = \frac{M}{\ell\sqrt{g}}\left(\sqrt{2gh} - \frac{Mg}{QB}\right)\left(\sqrt{2H} - \sqrt{2h}\right).$$

Because the mass can not be a negative quantity, it is necessary that $\frac{M}{QB} < \sqrt{\frac{2h}{g}}$ **.. 1 point**

c). $(u_y)_E = (u_y)_D = \sqrt{2gH}$,so that $u_E = \sqrt{2gH + u_x^2} = \left[g\frac{4H\left(\sqrt{H} - \sqrt{h}\right)^2 + \ell^2}{2\left(\sqrt{H} - \sqrt{h}\right)^2}\right]^{1/2}$ **.. 1 point**

Extra...1 point
Total...10 (ten) points

Problem 3

a).Ako je CD=x, AE=ℓ/2 i $ED = \ell\sqrt{3}/2$. Ukupno izduzenje je

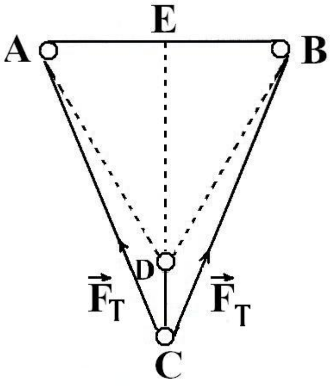
$\Delta\ell = 2\sqrt{(\ell/2)^2 + \left(\ell\sqrt{3}/2 + x\right)^2} + \ell - 2\ell$ **..1 p**

b). Izraz za $\Delta\ell$ može da se napiše u obliku $\Delta\ell = \ell\sqrt{1 + \left(\sqrt{3} + 2x/\ell\right)^2} - \ell \approx \ell + x\sqrt{3}$ (primedba**:** za $x = 0$ se dobija da je $\Delta\ell = \ell$)........1 p

c). U stanju ravnoteže važi $mg = 2k\ell\cos(\pi/6)$ pa je $k = mg/2\ell\cos 30^0 = mg/\ell\sqrt{3}$ **..................................... 1punct**

d). Restitutciona sila je $F = 2F_T\cos\beta - mg$, gde je $\beta = ugao(ECB)$

$F_T = k\Delta\ell$, sa $\cos\beta = \frac{x + \frac{\ell}{2}\sqrt{3}}{\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{\ell}{2}\sqrt{3} + x\right)^2}}$ daje,



$$F = 2k\left(\ell\sqrt{1+\left(\sqrt{3}+2x/\ell\right)^2}-\ell\right)\frac{x+\frac{\ell\sqrt{3}}{2}}{\sqrt{\left(\frac{\ell}{2}\right)^2+\left(x+\frac{\ell\sqrt{3}}{2}\right)^2}} - mg$$ **.. 2 p**

e). Za mala pomeranja, se dobija $F_T = k\Delta\ell = \frac{mg}{\ell\sqrt{3}}\left(\ell + x\sqrt{3}\right)$,

$\cos\beta = \frac{x+\ell\sqrt{3}/2}{\sqrt{(\ell/2)^2+\left(\ell\sqrt{3}/2+x\right)^2}} \approx \frac{\sqrt{3}+2x/\ell}{2}\frac{1}{\sqrt{1+x\sqrt{3}/\ell}} \approx \frac{\sqrt{3}}{2}+\frac{x}{4\ell}$, tako da je, na kraju

$$F = 2\frac{mg}{\ell\sqrt{3}}\left(x\sqrt{3}+\ell\right)\left(\frac{\sqrt{3}}{2}+\frac{x}{4\ell}\right) - mg = 2\frac{mg}{\ell\sqrt{3}}\left(\frac{\ell\sqrt{3}}{2}+\frac{7x}{4}\right) - mg = \frac{7mgx}{2\ell\sqrt{3}}.$$ **.......................... 2 p**

f). Efektivna krutost žice je $k_{eff} = \frac{7mg}{2\ell\sqrt{3}}$ **... 1 p**

g). Frekvencija malih oscilacija je $\omega = \sqrt{k_{eff}/m} = \sqrt{7g/2\ell\sqrt{3}}$ **..1 p**

Extra poen .... 1 p

TOTAL...10 poena

Provodni štap obrće se bez trenja oko vertikalne ose koja prolazi kroz centar horizontalnog provodnog prstena čiji je poluprečnik *l*, dok drugi kraj štapa klizi po tom prstenu pod dejstvom tangencijalne sile ***F*** (vidi sliku). Linije homogenog magnetnog polja indukcije ***B*** normalne su na ravan prstena. Izmedju ose i prstena priključen je izvor elektromotorne sile E preko otpornika *R*.

a) Izračunati moment sile magnetnog polja M_m, uzimajući u obzir da je $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
b) Naći ustaljenu ugaonu brzinu štapa ω_{const}.
c) Izraziti snagu ovako dobijenog strujnog generatora *P* u zavisnosti od ugaone brzine štapa.

Rešenje:

a) $|E_{ind}| = \frac{\Delta\Phi}{\Delta t} = B\frac{\Delta S}{\Delta t} = B\frac{l\cdot l\Delta\varphi}{2\Delta t} = \frac{Bl^2\omega\Delta t}{2\Delta t} \Rightarrow |E_{ind}| = \frac{Bl^2\omega}{2}$. (2 p.)

Polaritet E_{ind} poklapa se sa polaritetom E jer se indukovana Amperova sila, po Lencovom pravilu, suprotstavlja sili ***F*** $\Rightarrow I = \frac{1}{2}(E + E_{ind}) \Rightarrow I = \frac{1}{2R}(2E + Bl^2\omega)$. (2 p.)

Na element štapa dužine Δr deluje sila magnetnog polja $\Delta F_m = BI\Delta r$, čiji je moment: $\Delta M_m = r\Delta F_m = BIr\Delta r$. Izdelimo *l* na *n* jednakih delića, tako da je $\Delta r = l/n$. Razlika momenata na krajevima *i*-tog delića je $\Delta M_{mi} = \Delta F_m(i\Delta r)\cdot\Delta r$, dok će ukupni moment biti jednak: $M_m = \sum_{i=1}^{n} M_{mi} = \sum_{i=1}^{n} BI\cdot i\Delta r\cdot\Delta r = BI(\Delta r)^2\sum_{i=1}^{n} i = BI(\Delta r)^2\frac{n(n+1)}{2}$. Za $n\to\infty \Rightarrow \frac{n(n+1)}{2}\to\frac{n^2}{2}$

$\Rightarrow M_m = \frac{BI(n\Delta r)^2}{2} \Rightarrow M_m = \frac{BIl^2}{2}$. (2,5 p.)

Vidimo da je M_m isti kao da sila BlI deluje na sredinu štapa. (*Alternativno*: $dM_m = BIrdr$, $M_m = \int_0^l BIrdr = \frac{BIl^2}{2}$)

b) Ugaona brzina će biti jednaka nuli kada se izjednače momenti magnetne i mehaničke sile, pri konstantnoj struji: $\frac{1}{2}Bl^2 I_{const} = Fl \quad \Rightarrow \quad I_{const} = \frac{2F}{Bl}$, odnosno:

$\frac{2F}{Bl} = \frac{1}{R}$(E + E$_{ind}$). Odavde je E$_{ind} = \frac{2FR}{Bl} -$ E $\quad \Rightarrow \quad Bl^2\omega_{const} = \frac{2FR}{Bl} -$ E iz čega se konačno dobija:

$\omega_{const} = \frac{2}{Bl^2}(\frac{2FR}{Bl} - \text{E})$. (1,5 p.)

c) Snaga generatora: $P = M_m\omega$, $\quad P = \frac{BIl^2}{2}\omega = \frac{Bl^2\omega}{2}\cdot\frac{1}{2R}(2\text{E} + Bl^2\omega)$,

$P = \frac{Bl^2\omega}{2R}(\text{E} + \frac{Bl^2\omega}{2})$. (1 p.) (Bonus: 1 p.)

Second Problem. Transverse waves in strings

A uniform inextensible string of length L and total mass M, is suspended vertically and tapped at the top end so that a transverse impulse runs down it. At the same moment, a point-like body is released from rest, from the top of the strinng, and falls freely (on a parallel, vertical, direction).
a). How far from the bottom does the body pass the impulse ? Find the corresponding time instant.
b). Find the speed (velocity) of the body (v_b) and of the impulse in the string (v_i) at the instant when the body pass the impulse.
c). What is the ratio between the time interval in which the impulse in the string and the body traverse the distance L ?

Solution for the second problem

a). Posmatrajmo tačku koja je za x udaljena od donjeg kraja strune. Masa strune ispod ove tačke je očigledno Mx/L, tako da je napon zatezanja u ovoj tački Mgx/L.

Masa po jedinici dužine je M/L, pa je brzina transverzalnih talasa u tački x

$v = \sqrt{\frac{Mgx/L}{M/L}} = \sqrt{gx}$ **..1,5 points**

Vreme $T(x)$ potrebno impulse da predje rastojanje od $x' = L$ (gornji kraj) do x je onda

$T(x) = \int_x^L \frac{dx'}{\sqrt{gx'}} = 2\sqrt{\frac{x'}{g}}\Big|_x^L = \frac{2}{\sqrt{g}}\left(\sqrt{L} - \sqrt{x}\right)$**..1,5 points**

Vreme $t(x)$ potrebno tačkastom telu da pri padu, krenuvši iz mirovanja, predje rastojanje $L - x$ je

$t(x) = \sqrt{\frac{2(L-x)}{g}}$ **..1 point**

Računajući $T(x)$ i $t(x)$ da bi se dobila vrednost za koju telo sustiže impuls, dobijamo jednačinu $9x^2 - 10Lx + L^2 = 0$, sa rešenjem $x = L$ (za početni trenutak, kada su impuls i telo krenuli) i $x = L/9$ (rastojanje od donjeg kraja strune)**..1,5 points**

Sada se lako dobija

$T(L/9) = t(L/9) = \frac{4}{3}\sqrt{L/g}$. **..1,5 points**

b). $v_i = \frac{1}{3}\sqrt{Lg}$,

$v_b = \sqrt{2g(L-x)} = \sqrt{2g\frac{8L}{9}} = \frac{4}{3}\sqrt{Lg} = 4v_i$ **.......................................1 point**

c). $t_b = \sqrt{\frac{2L}{g}}$,

$t_i = 2\sqrt{\frac{L}{g}} = \sqrt{2}t_b$ so that $\frac{t_i}{t_b} = \sqrt{2} \approx 1{,}4142$. **.......................................1 point**

Extra..1 point

TOTAL...10 (ten) points