Second Class

First problem

The cyclic process 1-2-3-1, depicted on a p-V diagram (drawing) as in figure, is performed by a certain amount ($\nu = 1 kmol$) of an ideal monoatomic gas. The cycle is a triangle of equal legs (isosceles), with the basis 3-1 parallel with the OV axis. Knowing the absolute temperatures T_1, T_2 and T_3 from the (of the) points (states) 1,2 and 3 respectivelly, determine:

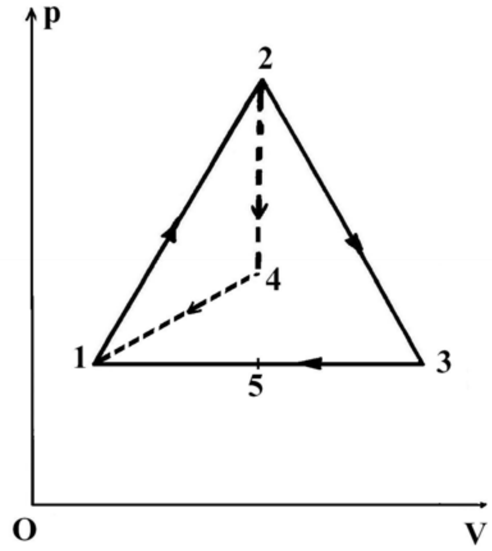
a). the work performed by the gas in the cyclic process 1-2-3-1;

b). the absolute temperature T_4 in the state 4, i.e. in the middle point of the triangle;

c). the efficiency of the cyclic process 1-2-4-1, having only rectilinear parts (formed with rectilinear segments 1-2, 2-4 and 4-1);

d). the absolute temperature of the state 5 with the parameters $V_5 = V_2$ and $p_5 = p_1 = p_3$.

Explanation : With the point 4 the area of the triangle 1231 is divided in three equal parts. It is known that the molar heat capacity at constant volume is $C_V = 3R/2$.



Solution of the first problem

a). The work performed by the gas is the area of the triangle $L_{1231} \equiv L = \frac{1}{2}(V_3 - V_1)(p_2 - p_1)$, where $V_2 = \frac{1}{2}(V_3 - V_1)$. With $pV = RT$ we find $L = \frac{1}{2}\left[R(T_1 - T_3) + p_2(V_3 - V_1)\right]$. Relation between the volumes may be writen as $\frac{2T_2}{p_2} = \frac{T_1}{p_1} + \frac{T_3}{p_3} = \frac{T_1 + T_3}{p_1}$ so that $p_2 = 2p_1 \frac{T_2}{T_1 + T_3}$. Finally we find

$L = \frac{R}{2}\frac{(T_3 - T_1)(2T_2 - T_1 - T_3)}{T_1 + T_3}$ **(*) .... 2 points**

b). With the observation that $3.\text{Aria}(1431) = \text{Aria}(1231)$ and using also the relation (*) for Area (1431) with T_4 instead of T_2, we obtain $T_4 = \frac{1}{3}(T_1 + T_2 + T_3)$ **..1,5 points**

c). In the cycle 1-2-4-1 the heat obtained by the gas is $Q_{12} = \Delta U_{12} + L_{12}$, where $\Delta U_{12} = C_V(T_2 - T_1) = \frac{3R}{2}(T_2 - T_1)$. and $L_{12} = \frac{L}{2} + p_1(V_2 - V_1) = \frac{L}{2} + \frac{R}{2}(T_3 - T_1)$. **......................1,5 points**

With the help of the relation (*), finally we find $L_{12}=\frac{R}{4}\frac{(T_3-T_1)(2T_2+T_1+T_3)}{T_3+T_1}$. **...1 point**

Using the relations for ΔU_{12} and L_{12} we have $Q_{12}=\frac{R}{4}.\frac{8T_2T_3+4T_1T_2-6T_1T_3-7T_1^2+T_3^2}{T_1+T_3}$ **.. 1 point**

The efficiency of the cycle is

$$\eta=\frac{L_{ef}}{Q_{pr}}=\frac{L}{3Q_{12}}=\frac{2(T_3-T_1)(2T_2-T_1-T_3)}{3(8T_2T_3+4T_1T_2-6T_1T_3-7T_1^2+T_3^2)}$$ **...1 point**

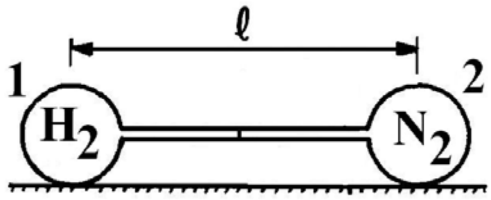
d). $T_5=\frac{p_5V_5}{R}=\frac{p_1V_2}{R}=\frac{p_1(V_3+V_1)}{2R}=\frac{1}{2}(T_3+T_1).$ **...1 point**

Extra...1 point

TOTAL...10 (ten) points

Second problem

On a horizontal table, perfectly smooth and long enough, lay two identical, spherical glass balloons, connected one to another through a very thin horizontal tube (the volume of which is to be neglected), having at its middle a separating membrane (see the figure). The distance between the centers of the balloons is $\ell=58cm$. In one balloon there is hydrogen while in the other, at the same temperature , but at a double pressure, there is nitrogen. If the separating membrane should break, by what distance the system would move along the tube ? The masses of the balloons and of their connecting tube will be neglected. The molar masses for hydrogen and nitrogen are $\mu_1=2kg/kmol$, respectively $\mu_2=28kg/kmol$.



Solution of the second problem

The gas masses are $m_1=\mu_1 pV/RT$ (for H_2) and $m_2=\mu_2 2pV/RT$ (for N_2) **.............................2 points**

Let us denote by x the distance between the middle of the connecting tube and the center of mass for the system. We can write $\left(\frac{\ell}{2}+x\right)m_1=\left(\frac{\ell}{2}-x\right)m_2$ **..2 points**

Extractig x we find $x=\frac{1}{2}\frac{m_2-m_1}{m_2+m_1}=\frac{1}{2}\frac{2\mu_2-\mu_1}{2\mu_2+\mu_1}=27$ cm. **...2 points**

When the separating membrane disappear by break, the gas content into system become homogeneous and the center of mass are located at the middle of the connecting tube...**1 point**

In the horizontal direction this system is a isolated mechanical system . This is the reason that in the table's frame of reference the center of mass rest at the initial position, i.e. the the two ballons

together with connecting tube go to the right on the distance x=27cm**...2 points**
Extra.. 1 point
TOTAL...10(ten) points

Problem 3

Problem 2. In this problem, the starting pressure P_a and volume V_a of an ideal gas in state a, are given. The ratio $R_V = V_c / V_a > 1$ of the volumes of the states c and a is given. Finally a constant γ=5/3 is given. You do not know how many moles of the gas are present.

(1) In the first of four steps, *a* to *b* , an ideal gas is compressed from V_a to V_b while no heat is allowed to flow into or out of the system. The compression of the gas raises the temperature from an initial temperature T_1 and to a final temperature T_2. During this process the quantity $PV^{\gamma} = const.$

a) What is the pressure P_b and volume V_b of the state b of the gas after the compression is finished?

b) What is the change in internal energy of the gas during this change of state?

c) What is the work done by the gas during this compression?

(2) The gas is now allowed to expand isothermally from *b* to *c* , from volume V_b to volume V_c.

d) Express the work done by the gas in this process A_{cb} and the amount of heat Q_{cb} that must be added from the heat source at T_2 in terms of P_a, V_a, T_2, T_1, and V_c. Is this heat positive or negative? Explain whether it is added to the system or removed.

e) What is the pressure P_c of the gas after the expansion is finished?

(3) When the gas has reached point *c* is expands from V_c to V_d while no heat is allowed to flow into or out of the system. The expansion of the gas lowers the temperature and pressure from an initial temperature T_2 to a final temperature T_1. During this process the quantity $PV^{\gamma} = const.$

f) What is the pressure P_d and volume V_d of the state d of the gas after the expansion is finished?

g) What is the change in internal energy of the gas during this change of state?

h) What is the work done by the gas during this expansion?

(4) The gas is now compressed isothermally from *d* to *a* at constant T_1 from volume V_d back to V_a.

i) Find the work done by the system on the surroundings A_{ad} and the amount of heat Q_{ad} that flows between the system and the surroundings. Are these quantities positive or negative? Explain whether heat is added to the system or removed from the heat source at T_1.

Total Cycle:

j) What is the total work A_{cycle} done by the gas during this cycle?

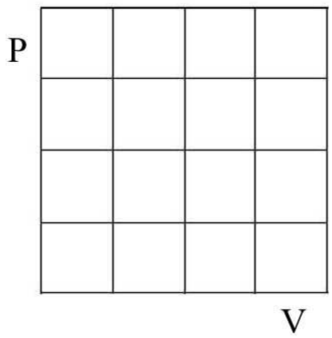
k) What is the total heat Q_{cycle} (from T_2) drawn from the higher temperature heat source during this cycle?

l) What is the efficiency of this cycle $\varepsilon_{max} = A_{cycle} / Q_{cycle}$ (from T_2)?

m) Knowing the steps (1) – (4) above sketch the path of the cycle on a *PV* plot on the graph below. Label all appropriate points.

Answers:

***** (1) *****



a) According to the ideal gas law, $P_b V_b = n_m R T_2$ and $P_a V_a = n\ R T_1$ so

$$P_b V_b = P_a V_a \frac{T_2}{T_1}.$$

So the pressure

$$P_b = P_a \frac{V_a}{V_b} \frac{T_2}{T_1}.$$

The compression satisfies $P_b V_b^{\gamma} = P_a V_a^{\gamma}$ so using the above result for pressure P_b, we get

$$P_b V_b^{\gamma} = P_a \frac{V_a}{V_b} \frac{T_2}{T_1} V_b^{\gamma} = P_a V_a^{\gamma}.$$

This becomes using $\gamma = 5/3$

$$V_b^{2/3} = \frac{T_1}{T_2} V_a^{2/3}.$$

The volume V_b is then

$$V_b = \left(\frac{T_1}{T_2}\right)^{3/2} V_a.$$

Thus the ratio of the volumes is

$$\frac{V_b}{V_a} = \left(\frac{T_1}{T_2}\right)^{3/2}.$$

So the pressure P is

$$P_{\mathrm{b}} = P_{\mathrm{a}} \left(\frac{T_1}{T_2} \right)^{5/2} .$$

b) The change in internal energy is

$$U_{\mathrm{b}} - U_{\mathrm{a}} = \frac{3}{2} n_m R \Delta T = \frac{3}{2} P_{\mathrm{a}} V_{\mathrm{a}} \frac{(T_2 - T_1)}{T_1} .$$

c) Since no heat is exchanged $Q_{\mathrm{ba}}=0$

$$U_{\mathrm{b}} - U_{\mathrm{a}} = -A_{\mathrm{ba}} + Q_{\mathrm{ba}} = -A_{\mathrm{ba}} = \frac{3}{2} P_{\mathrm{a}} V_{\mathrm{a}} \frac{(T_2 - T_1)}{T_1} .$$

So

$$A_{\mathrm{ba}} = -\frac{3}{2} P_{\mathrm{a}} V_{\mathrm{a}} \frac{(T_2 - T_1)}{T_1} < 0 .$$

The surroundings do work compressing the gas.

***** (2) *****

d) This is an isothermal expansion so the temperature does not change $\Delta T=0$. Thus the internal energy is constant,

$$U_{\mathrm{c}} - U_{\mathrm{b}} = \frac{3}{2} n_m R \Delta T = 0 .$$

The gas does work on the surroundings because it is expanding. The pressure is not constant during this expansion. Since the gas is expanding by an isothermal process, the Ideal Gas Law relates the pressure and volume variation according to

$$P = \frac{n_m R T}{V} .$$

Therefore the work done by the gas on the surroundings is the integral

$$A_{\mathrm{cb}} = n_m R T_2 \int_{V_{\mathrm{b}}}^{V_{\mathrm{c}}} \frac{\mathrm{d}V}{V} = n_m R T_2 \ln\left(\frac{V_{\mathrm{c}}}{V_{\mathrm{b}}} \right) .$$

Using the result for the volume V from part a)

$$V_{\mathrm{b}} = \left(\frac{T_1}{T_2} \right)^{3/2} V_{\mathrm{a}} ,$$

the work is

$$A_{\text{cb}} = n_m R T_2 \ln\left(\left(\frac{T_1}{T_2}\right)^{3/2} \frac{V_{\text{c}}}{V_{\text{a}}}\right).$$

Recall that the volumes are related according to $R_V = V_{\text{c}}/V_{\text{a}} > 0$ and $n_m R = P_{\text{a}} V_{\text{a}}/T_1$ so the work done is positive and given by

$$A_{\text{cb}} = P_{\text{a}} V_{\text{a}} \frac{T_2}{T_1} \ln\left(\left(\frac{T_1}{T_2}\right)^{3/2} R_V\right) > 0 .$$

From The First Law of Thermodynamics,

$$U_{\text{c}} - U_{\text{b}} = -A_{\text{cb}} + Q_{\text{cb}} = 0 .$$

Thus the heat that flows into the system from the heat source at temperature T_2 is equal to the work done by the expanding gas.

$$Q_{\text{cb}} = A_{\text{cb}} = P_{\text{a}} V_{\text{a}} \frac{T_2}{T_1} \ln\left(\left(\frac{T_1}{T_2}\right)^{3/2} R_V\right) > 0 .$$

Note that this heat flow must flow from the higher temperature heat source into the system because as the gas expands it should lose internal energy and would decrease its temperature unless heat flows into the system keeping the internal energy and hence the temperature constant.

e) $P_{\text{c}} V_{\text{c}} = n_m R T_2 = P_{\text{a}} V_{\text{a}} \dfrac{T_2}{T_1}$.

Thus

$$P_{\text{c}} = \frac{P_{\text{a}}}{R_V} \frac{T_2}{T_1} .$$

***** (3) *****

f) This calculation is identical to part a), with state d replacing state a, and state c replacing state b. So the volume V is then

$$V_{\text{c}} = \left(\frac{T_1}{T_2}\right)^{3/2} V_{\text{d}} .$$

Thus the ratio of the volumes is

$$\frac{V_c}{V_d} = \left(\frac{T_1}{T_2}\right)^{3/2}.$$

So the pressure P_c is

$$P_c = P_d \left(\frac{T_2}{T_1}\right)^{5/2},$$

hence

$$P_d = P_c \left(\frac{T_1}{T_2}\right)^{5/2}.$$

g) The decrease in the internal energy is due to the temperature decrease of the ideal gas during expansion

$$U_d - U_c = \frac{3}{2} P_a V_a \frac{(T_1 - T_2)}{T_1}.$$

h) Since no heat is exchanged $Q_{dc}=0$

$$U_d - U_c = -A_{dc} + Q_{dc} = -A_{dc} = \frac{3}{2} P_a V_a \frac{(T_1 - T_2)}{T_1}.$$

So

$$A_{dc} = \frac{3}{2} P_a V_a \frac{(T_2 - T_1)}{T_1} > 0.$$

The gas does work on the surroundings since the gas is expanding.

***** (4) *****

i) When the gas undergoes compression it will increase its internal energy but heat flows out of the system maintaining constant internal energy, $\Delta U = 0$ and hence the compression is isothermal. The calculation of the work and heat is similar to step (2) except the temperature is held at T_1. The work done by the system on the surroundings is negative and is given by the integral

$$A_{ad} = n_m R T_1 \int_{V_d}^{V_a} \frac{dV}{V} = n_m R T_1 \ln\left(\frac{V_a}{V_d}\right) = P_a V_a \ln\left(\frac{V_a}{V_d}\right).$$

From part f) the volume $V_d = \left(\frac{T_2}{T_1}\right)^{3/2} V_c$ so the work done is

$$A_{\rm ad} = P_{\rm a}V_{\rm a}\ln\left(\frac{V_{\rm a}}{V_{\rm d}}\right) = P_{\rm a}V_{\rm a}\ln\left(\frac{V_{\rm a}}{\left(\frac{T_2}{T_1}\right)^{3/2}V_{\rm c}}\right) = -P_{\rm a}V_{\rm a}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right)$$

According to the First Law this is equal to the heat that flows into the system which is also negative which means that it actually flows out of the system into the surroundings at temperature T_1,

$$Q_{\rm ad} = A_{\rm ad} = -P_{\rm a}V_{\rm a}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right).$$

Total Cycle:

j) The work done by the heat engine on the surroundings during the cycle is positive and given by

$$A_{\rm cycle} = P_{\rm a}V_{\rm a}\frac{T_2}{T_1}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right) - P_{\rm a}V_{\rm a}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right) = P_{\rm a}V_{\rm a}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right)\left(\frac{T_2}{T_1}-1\right).$$

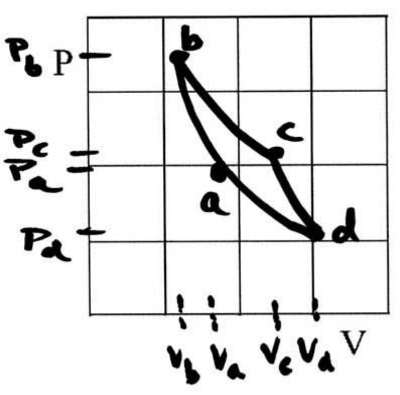
k) The heat that flowed from the higher temperature heat source T_2 occurred during step (2) $\mathrm{b}\rightarrow\mathrm{c}$ isothermal expansion,

$$Q_{\rm cycle}\ \text{taken from heat source at } T_2 = P_{\rm a}V_{\rm a}\frac{T_2}{T_1}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right).$$

l) The efficiency is given by ratio of the work done divided by the heat flowing into the system from the higher temperature heat source

$$\varepsilon_{\max} = \frac{W_{\rm cycle}}{Q_{\rm cycle}} = \frac{P_{\rm a}V_{\rm a}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right)\left(\frac{T_2}{T_1}-1\right)}{P_{\rm a}V_{\rm a}\frac{T_2}{T_1}\ln\left(\left(\frac{T_2}{T_1}\right)^{3/2}R_V\right)} = \frac{\left(\frac{T_2}{T_1}-1\right)}{\frac{T_2}{T_1}} = \frac{T_2-T_1}{T_2} = \frac{\Delta T}{T_2}.$$

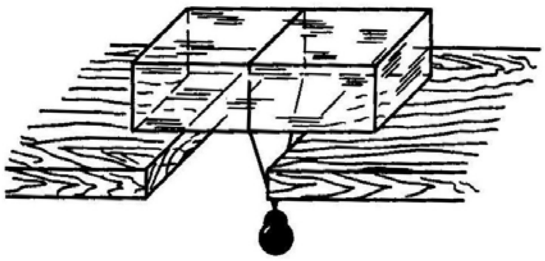
m)



Problem 4

A). To determine the quality of thermal insulation of a Dewar vessel, it is filled with ice at $0^{0}C$. After 24 hours, a quantity of 42 g of ice have melted. Usually liquid nitrogen at 78 K is kept in this flask. Assuming the quantity of heat entering the flask to be proportional to the difference in the internal and the external temperatures of the vessel, find the amount of liquid nitrogen that is going to evaporate in 24 hours. The ambient temperature is 20^{0}C. The heat of vapourization of liquid nitrogen at normal pressure is $L=1,8.10^{5}$J/kg.The specific heat of fuision for ice is $\lambda=3,4.10^{5}$ J/kg.

B). If you sling a thin wire loop around a block of ice and attach to it a weight of several kilograms then after some time the wire will pass through the block of ice, but the block remains intact (see figure). Explain this phenomenon. The ambient temperature is -2^{0}C and the atmospheric pressure is normal.



Solution

A). (6 points).The heat flowing to the Dewar vessel is $Q=\alpha(T_{air}-T)$, where α is a certain coefficient, and T is the temperature inside the flask**..1 point**

For ice and liquid nitrogen we obtain the ratio $\frac{Q_1}{Q_2}=\frac{T_{air}-T_1}{T_{air}-T_2}$ **...1,5 points**

But, for nitrogen $Q_2=m_2L$, where L is its heat of evaporation and for ice $Q_1=m_1\lambda$ **…………1 point**

Hence $\frac{m_1\lambda}{m_2L}=\frac{T_{air}-T_1}{T_{air}-T_2}$, from which $m_2=\frac{m_1\lambda(T_{air}-T_2)}{L(T_{air}-T_1)}$ **..1,5 points**

Numerically $m_2=853$ grams**...1 point**

B).(3 points). Ice melts under the pressure of the wire, and the wire sinks; the water formed above the wire immediately freezes again.

Extra...1 point

Total...10 (ten) points.